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Abstract

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PHYSICS

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ON THE MOTION OF THE INTERFACE BETWEEN TWO PHASES IN SUPERCONDUCTING THIN FILMS

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For superconductors of the second kind, which include thin films (the film thickness d is much smaller than the penetration depth of the magnetic field into the superconductor δ), the Maxwell-London equations ([1]) are valid:

$$\operatorname{rot}\mathbf{H} = \frac{\varepsilon}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}, \qquad \operatorname{rot}\mathbf{E} = -\frac{\mu}{c}\frac{\partial \mathbf{H}}{\partial t}, \qquad \operatorname{div}\mathbf{E} = \frac{4\pi}{\varepsilon}\rho, \qquad \operatorname{div}\mathbf{H} = 0,$$

$$\rho_s + \rho_n = \rho, \qquad \mathbf{j}_s + \mathbf{j}_n = \mathbf{j}, \qquad \mathbf{j}_n = \sigma\mathbf{E}, \tag{1}$$

$$\mathbf{E} = \frac{d}{dt}(\lambda \mathbf{j}_s), \qquad \lambda = \frac{m}{n_s e^2} \geq \frac{4\pi\mu\delta^2}{c^2} = \lambda_0,$$

where $\mathbf{E}$ is the electric-field strength; $\mathbf{j}$ is the current-density vector; σ is the normal conductivity, finite at all temperatures; λ is the London constant, depending on the absolute temperature T and on the magnetic-field strength $\mathbf{H}$; n_s is the concentration of superconducting electrons (Cooper pairs); e is the electron charge; ρ is the charge density; m is the electron mass; μ is the magnetic permeability; c is the speed of light in vacuum; ε is the dielectric permittivity. The subscripts s and n refer, respectively, to the superconducting and normal states. In what follows it is assumed that $\rho = 0$; $\operatorname{div}\mathbf{j}_n = 0$; $\operatorname{div}\mathbf{j}_s = 0$.

Superconducting plane films permit the introduction of current functions ([2]) $\psi_s(x,y,t)$, $\psi_n(x,y,t)$, $\psi = \psi_s + \psi_n$, with respect to the coordinates x, y in the plane of the film (the z-axis is directed normal to the film):

$$j_{sx} = \partial\psi_s/\partial y, \qquad j_{sy} = -\partial\psi_s/\partial x,$$

$$j_{nx} = \partial\psi_n/\partial y, \qquad j_{ny} = -\partial\psi_n/\partial x, \tag{2}$$

$$j_x = \partial\psi/\partial y, \qquad j_y = -\partial\psi/\partial x.$$

Fig. 1. $x < x_1$ –superconducting phase, $x > x_2$ –normal phase; x_1x_2 – intermediate state

The Maxwell-London equations for such films take the form

$$\Delta\psi_n - \frac{1}{\sigma}\frac{d\sigma}{dT}(\nabla T \cdot \nabla\psi_n) = \frac{4\pi\mu\sigma}{c^2}\left(\frac{\partial\psi}{\partial t} + \frac{\varepsilon}{4\pi\sigma}\frac{\partial^2\psi_n}{\partial t^2}\right),$$

$$\Delta\psi_s + \frac{1}{\lambda}\left(\frac{\partial\lambda}{\partial T}\nabla T + \frac{\partial\lambda}{\partial\psi}\nabla\psi\right)\nabla\psi_s = -\frac{4\pi\mu}{c^2\lambda}\left(\psi + \frac{\varepsilon}{4\pi\sigma}\frac{\partial\psi_n}{\partial t}\right), \tag{3}$$

where Δ is the two-dimensional Laplace operator; $\psi = \frac{c}{4\pi}H_z$, $H_x = H_y = 0$. The boundary conditions on the contour of the film are $\partial\psi/\partial l = \partial\psi_s/\partial l = \partial\psi_n/\partial l = 0$ (l is the direction of the contour).

Let x_1x_2 be the layer separating the superconducting region from the normal one (Fig. 1). Suppose that the fields are quasistationary, the problem is one-dimensional (x, t), and $\lambda = \lambda(\mathbf{H})$ (the supercritical region with respect to T). Under these assumptions equations (3) take the form

$$\Delta\psi_n = -\frac{4\pi\mu\sigma}{c^2}\frac{\partial\psi}{\partial t},$$

$$\lambda\Delta\psi_s + \frac{d\lambda}{d\psi}\frac{\partial\psi}{\partial x}\frac{\partial\psi_s}{\partial x} = -\frac{4\pi\mu}{c^2}\psi. \tag{4}$$

Using the relation $\psi = \psi_s + \psi_n$, one can transform (4) to a form containing only ψ_s, or only ψ_n, and their derivatives.

Integrating (4) with respect to x, we find

$$-\frac{\partial\psi_n}{\partial x} = j_n = -\frac{4\pi\mu\sigma}{c^2}\frac{\partial}{\partial t}\int_{-\infty}^{x}\psi\,dx. \tag{5}$$

Multiplying the first equation (4) by λ and adding it to the second, we obtain

$$\lambda\Delta\psi + \frac{\partial\lambda}{\partial x}\frac{\partial\psi_s}{\partial x} = -\frac{4\pi\mu\sigma}{c^2}\left(\lambda\frac{\partial\psi}{\partial t} + \frac{1}{\sigma}\psi\right). \tag{6}$$

Further,

$$\frac{\partial\psi_s}{\partial x} = \frac{\partial\psi}{\partial x} - \frac{\partial\psi_n}{\partial x} = \frac{\partial\psi}{\partial x} - \frac{4\pi\mu\sigma}{c^2}\frac{\partial}{\partial t}\int_{-\infty}^{x}\psi\, dx. \tag{7}$$

Substituting (7) into (6), we find

$$\Delta\psi + \frac{\partial\ln\lambda}{\partial x}\left(\frac{\partial\psi}{\partial x} - \frac{4\pi\mu\sigma}{c^2}\frac{\partial}{\partial t}\int_{-\infty}^{x}\psi\, dx\right) = -\frac{4\pi\mu\sigma}{c^2}\left(\frac{\partial\psi}{\partial t} + \frac{1}{\sigma\lambda}\psi\right). \tag{8}$$

We restrict ourselves to stationary solutions of the form $\psi(x,t) = \psi(x - vt)$. Figure 1 shows the dependence of H on x. In the linear approximation,

$$\left(\frac{\partial H}{\partial x}\right)_1 = \frac{H_{k2} - H_{k1}}{x_2 - x_1} = \frac{H_{k2} - H_{k1}}{-v\tau}, \tag{9}$$

where v is the propagation velocity of the normal phase, and τ is the time of the phase transition. The point x_1 corresponds to the start of the reaction and to the magnetic field H_{k1} (the first critical field), while the point x_2 corresponds to the end of the reaction and to the magnetic field H_{k2} (the second critical field).

Equation (8) in the region to the left of x_1 takes the form

$$\frac{\partial^2\psi}{\partial x^2} - \frac{4\pi\mu\sigma}{c^2}\left(\frac{\partial\psi}{\partial t} + \frac{1}{\sigma\lambda_0}\psi\right) = \ddot{\psi} + \alpha\dot{\psi} + \beta\psi = 0, \tag{10}$$

where $\dot{\psi} = d\psi/dz$, $\ddot{\psi} = d^2\psi/dz^2$, $\partial\psi/\partial t = -v\, d\psi/dz$, $\alpha = 4\pi\mu\sigma v/c^2$, $\beta = -4\pi\mu/\lambda_0 c^2$, $z = x - vt$.

The characteristic equation for (10) gives the roots

$$2p_1 = -\alpha + \sqrt{\alpha^2 - 4\beta} > 0, \qquad 2p_2 = -\alpha - \sqrt{\alpha^2 - 4\beta} < 0.$$

The general solution of equation (10) is $\psi = C_1 e^{p_1 z} + C_2 e^{p_2 z}$. Since for points to the left of x_1 the coordinate $x < 0$, and at $x = -\infty$, $\psi = 0$, it follows that $C_2 = 0$ and $\psi = C_1 e^{p_1 z} = \psi_{k1} e^{p_1 z}$, where

$$\psi_{k1} = \frac{c}{4\pi}H_{k1}, \qquad \psi = \frac{c}{4\pi}H.$$

Differentiating the solution found with respect to x, we obtain

$$2\frac{\partial H_{k1}}{\partial x} = 2H_{k1}p_1 = H_{k1}\left(-\frac{4\pi\mu\sigma}{c^2}v + \sqrt{\left(\frac{4\pi\mu\sigma}{c^2}\right)^2 v^2 + \frac{16\pi\mu}{c^2\lambda_0}}\right). \tag{11}$$

Squaring the last relation, using (9), and setting $H_{k2} = \eta H_{k1}$, we find

$$v = \frac{(1-\eta)c}{2}\sqrt{\frac{\lambda_0}{\pi\mu\tau(\tau+\delta\lambda_0(\eta-1))}}. \tag{12}$$

The quantity v does not exceed the velocity of propagation of phonon perturbations and is of the order of the speed of sound. In the phase transition of a type-II superconductor from the superconducting to the normal state, in contrast to type-I superconductors, the negative sign of the surface energy makes equilibrium of the normal and superconducting phases impossible. The normal phase propagates through the superconducting phase with velocity v.

To calculate v from formula (12), it is necessary to know the phase-transition time τ. To calculate τ, let us consider the motion of the normal phase as the motion of the surface $H = H_{k1}$. The boundary conditions on such a surface will be

$$p_1^+ = \left(\frac{\partial H}{\partial x}\right)_1^+ = -\frac{4\pi}{c} j_1^+ = -\frac{4\pi}{c}(j_n^+ + j_s^+)_1, \tag{13}$$

where the sign + indicates the value of the corresponding physical quantities on the surface $H = H_{k1}$ from the right (Fig. 1).

Relations (13) follow from the Maxwell–London equations (1). Differentiating (13) with respect to t, we obtain

$$\frac{\partial}{\partial t}\left(\frac{\partial H}{\partial x}\right)_1^+ = -v\left(\frac{\partial^2 H}{\partial x^2}\right)_1^+ = -\frac{4\pi}{c}\left(\sigma\frac{\partial E_1^+}{\partial t} + \frac{\partial j_{s1}^+}{\partial t}\right) = -\frac{4\pi}{c}\left[-\sigma v\left(\frac{\partial E}{\partial x}\right)_1^+ + \frac{E_1^+}{\lambda_0}\right]$$

$$= -\frac{4\pi}{c}\left[\frac{\mu\sigma v^2}{c}\left(\frac{\partial H}{\partial x}\right)_1^+ + \frac{E_1^+}{\lambda_0}\right] = -\frac{4\pi}{c}\left(-\frac{4\pi\mu\sigma^2 v^2}{c^2}E_1^+ + \frac{E_1^+}{\lambda_0}\right). \tag{14}$$

In (14), $(\partial E/\partial x)_1^+$ is expressed in terms of $\partial H_1^+/\partial t$ from the second equation (1), and the operator $\partial/\partial t$ is replaced by $-v\,\partial/\partial x$ (because on the surface $H = H_{k1}$ the value of the magnetic field is conserved in time). Considering the surface $H = H_{k1}$ as a surface of strong discontinuity between the purely superconducting phase (to the left of x_1 in Fig. 1) and the remaining part of the film, one can establish a relation between E_1^+ and H_{k1}^+. Neglecting the penetration depth δ of the magnetic field into the superconducting part of the film in comparison with the characteristic size (diameter) of the film, we obtain ([3],[4])

$$E_1^+ = \frac{v}{c}\,\mu H_{k1}. \tag{15}$$

Substituting E_1^+ from (15) into (14), noting that $(\partial^2 H/\partial x^2)_1^+ = (p_1^+)^2 H_{k1}$, and canceling by $v \neq 0$ and $H_{k1} \neq 0$, we find

$$(p_1^+)^2 = 4\pi\mu/\lambda_0 c^2 - (4\pi\mu\sigma/c^2)^2 v^2. \tag{16}$$

On the other hand, by continuity $p_1^+ = p_1^-$, where p_1^- is the value of p on the surface $H = H_{k1}$ from the left (Fig. 1). From (11) we have

$$(p_1^-) = \frac{1}{4}\left(-\frac{4\pi\mu\sigma v}{c^2} + \sqrt{\left(\frac{4\pi\mu\sigma}{c^2}\right)^2 v^2 + \frac{16\pi\mu}{\lambda_0 c^2}}\right). \tag{17}$$

Equating p_1^+ from (16) and p_1^- from (17), we obtain

$$v = -\frac{c}{2\sigma\sqrt{2\pi\mu\lambda_0}} = -\frac{ec}{2\sigma}\sqrt{\frac{n}{2\pi\mu m}}, \tag{18}$$

where n is the total concentration of electrons in the metal (in 1 cm^3). Formula (18) contains no explicit magnetic fields H_{k1} and H_{k2}. Substituting into (18) $\sigma =$

$= 10^{20},\ c = 3 \cdot 10^{10},\ e = 4.8 \cdot 10^{-10},\ n \sim 10^{22},\ \mu \approx 1,\ m \cong 10^{-27}$, we obtain $v \sim 10^5$ cm/sec.

Comparing (18) with (12), we find

$$\tau = \delta\lambda_0(\eta - 1) = \frac{\delta m}{ne^2}\left(\frac{H_{k2}}{H_{k1}} - 1\right). \tag{19}$$

For type-II superconductors $\eta = H_{k2}/H_{k1} \approx 10^3 \gg 1$, and therefore one may write

$$\tau = \frac{\delta m}{ne^2}\frac{H_{k2}}{H_{k1}}. \tag{20}$$

For the values of the physical quantities given above, $\tau \sim 10^{-7}$ sec. Depending on σ, n, η, for various superconductors τ, calculated from (20), varies within the range 10^{-6}–10^{-8} sec. The width of the reaction zone $x_2 - x_1 = v\tau$ fluctuates within the range 10^{-1}–10^{-3} cm. In any case $x_2 - x_1 \gg \xi$, where ξ is the parameter of the Cooper correlation. The condition $x_2 - x_1 \gg \xi$ is in full agreement with the requirement for type-II superconductors and with the admissibility of the Maxwell-London equations.

The result obtained on the motion of a disturbance in the form of a normal phase in a type-II superconductor must be taken into account in the design of thin-film cryotrons. Although the film is stable with respect to a disturbance of the magnetic field, during the magnetic relaxation time $\tau_H = 4\pi\mu\sigma : c^2\gamma_1$, where μ is the magnetic permeability, c is the speed of light in vacuum, and γ_1 is the smallest eigenvalue of the equation $\Delta\psi + \gamma\psi = 0$ with boundary conditions $\psi = 0$ on the contour of the film l ([2]), the normal phase will have time to propagate over considerable distances. For a film with conductivity $\sigma \cong 10^{20}$, thickness 0.3 μ, width 0.03 mm, and length 2 mm, $\tau \cong 10^{-7}$ sec, and assuming $v = 10^5$ cm/sec, we obtain the region of expansion of the normal phase $v\tau_H = 0.1$ mm, which considerably exceeds the width of the film.

The phase-transition time τ, determined by (20), limits the speed of response of thin-film cryotrons. Technical realization of considerable speed is impossible if the switching time is less than the phase-transition time τ. Consequently, the problem of increasing the speed of response of a cryotron reduces to decreasing the ratio H_{k2}/H_{k1}, i.e., to obtaining optimal magnetic properties of the film (provided that the reserves for decreasing τ by means of σ and n have been used).

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REFERENCES CITED

1. P. de Gennes, *Superconductivity of Metals and Alloys*, Moscow, 1968.

2. V. I. Skobelkin, E. A. Zhilkov, V. I. Puchkov, *FTT*, **11**, 10 (1969).

3. L. D. Landau, E. M. Lifshitz, *Electrodynamics of Continuous Media*, 1957, p. 227.

4. A. B. Pippard, *Phil. Mag.*, **41**, No. 314 (1950).

Note: Figure translations are in progress. See original paper for figures.