A MODEL OF THE ACCUMULATION AND REDISTRIBUTION OF THERMAL ENERGY IN AN ATMOSPHERIC LAYER

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A MODEL OF THE ACCUMULATION AND REDISTRIBUTION OF THERMAL ENERGY IN AN ATMOSPHERIC LAYER

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Let us consider an atmospheric layer above an underlying surface, in which the intensity of heat propagation is controlled by the mechanism of free convection. We shall take into account the interdependence of the temperature stratification of the medium and the intensity of convective flows. Convective flows are blocked until the mean temperature gradient γ in a layer of thickness H is less than a certain value γ^*, at which the critical value of the Rayleigh number is attained, $\mathrm{Ra} \sim \gamma H^4$. During this period, heat from the underlying surface is propagated by thermal conduction with a certain effective coefficient λ; the temperature gradient increases and thermal energy accumulates in the layer. When $\gamma \geq \gamma^*$, the blocking is removed and convective flows arise; the effective thermal conductivity of the layer increases instantaneously, and the thermal energy of the layer is discharged. Since, in the process of convection, heat transfer is carried out by moving masses of fluid, inertial forces appear, and the convective discharge will continue for some time after the gradient has fallen below the value γ^*. Periodic equalization of temperature in a layer of air was indeed observed by A. A. Skvortsov ([1]).

Fig. 1. Formation of a thermal in a layer of aniline heated from below: *a*, *b*—the layer during the period of heat accumulation; *v*, *g*—formation of a thermal during convective discharge

The processes of gradual heat accumulation in a layer and its violent convective discharge are observed in a laboratory model consisting of a layer of aniline covered with water ([2]). Heat supplied from below is blocked by the aniline layer and accumulates in it up to a certain limit (Fig. 1*a*, *b*), after which convective discharge occurs and the aniline thermal is ejected upward (Fig. 1*v*, *g*). Periodic destruction and restoration of convective layers in the atmosphere was observed and photographically recorded by Skvortsov ([1]).

Fig. 2. Scheme of variation of the coefficient of temperature conductivity. τ—period of self-oscillations, t_n—accumulation time, t_p—discharge time

Fig. 3

Figure 1: Fig. 3

Thus, the problem reduces to considering a heat-exchange system with convective blocking. A mathematical model of such a system can be constructed on the basis of the heat-conduction equation

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left[k(z,t)\frac{\partial T}{\partial z}\right],$$

if the character of the variation of the effective coefficient of temperature conductivity $k(z,t)$ is taken into account when the accumulation period is replaced by the discharge period.

(Fig. 2). Owing to this, it becomes possible to consider these periods separately. A similar procedure is used in the case of frictional self-oscillations ([3,4]). In this connection, it is essential that for each of the periods considered one has to deal with a linear equation. The nonlinear factor, as it were, switches the system from one linear regime to another. The application of known solutions of the heat-conduction equation ([5]) gives, in our case, simple relations characterizing the periods of accumulation and discharge of thermal energy in the near-surface layer.

Accumulation period. Consider a semi-infinite medium, isothermal at the initial instant, with a constant influx of heat f_0 from the underlying surface $z = 0$.

The heat flux at level z

$$f(z,t) = f_0 \operatorname{erfc}\tilde{z}, \quad \text{where } \tilde{z} = z/h,$$
$$h = 2\sqrt{\chi t}; \qquad \operatorname{erfc}\tilde{z} = 1 - \operatorname{erf}\tilde{z};$$

χ is the coefficient of effective thermal diffusivity. The deviation $T(z,t)$ from the initial temperature T_0 of the isothermal layer is determined by the formula:

$$T(z,t) = T_0\left[\exp(-\tilde{z}^2) - \sqrt{\pi}\tilde{z}\operatorname{erfc}\tilde{z}\right], \tag{1}$$

where $T_0 = T(0,t) = \pi^{-1/2}\varphi_0 h$ is the deviation of the surface temperature; $\varphi_0 = f_0/\lambda$, with λ the effective thermal conductivity of the air before the onset of convection.

Fig. 3. Change in the temperature stratification of the layer. $1, 2, 3$—temperature distribution in the process of heat accumulation for successive times $t = t_\text{n}/25$, $t_\text{n}/4$, and t_n, respectively (t_n is the instant at which the convective stability of the layer is lost); $3'$ approximates curve 3, referring to the instant t_n;

curves $4, 5, 6$ give the temperature distribution during discharge for successive times $t = 0.1t_{\text{p}},\ 0.2t_{\text{p}}$, and t_{p}, respectively (t_{p} is the discharge time of the layer).

In Fig. 3 the temperature curves $1, 2, 3$ are shown, calculated for successive times $t = t_{\text{n}}/25,\ t_{\text{n}}/4,\ t_{\text{n}}$, where t_{n} is the accumulation time, i.e., the interval of time up to the onset of convective motions in the layer. As t increases, T_0 increases and the segment of the curve $T(z,t)$ becomes steeper in a layer of a certain prescribed thickness H. The growth of the temperature gradient γ at each height z_i with time may be inferred from the expression

$$[\gamma(z,t)/\varphi_0]_{z=z_i} = \operatorname{erfc}\tilde{z}_i.$$

At a certain value $\gamma^* \sim T_0^*/H$ the layer loses stability, and the accumulation period is replaced by the discharge period. Consequently, the accumulation time t_{n} can be determined from the condition

$$\pi^{-1/2}\varphi_0\, 2\sqrt{\chi t_{\text{n}}} = T_0^*.$$

Discharge period. We approximate the distribution $T(z, t_{\text{n}})$, which at this stage serves as the initial one, by some simple function, for example a linear one,

$$T(z,0) = T_0^* - \gamma^* z,$$

where $T_0^* = T(0, T_{\text{n}})$ and $\gamma^* = T_0^*/H$. If the influx of heat through the lower boundary of the layer is neglected, i.e., if one sets $(\partial T/\partial z) = 0$ at $z = 0$, and takes the instant t_{n} as the initial one, then $T(z,t)$ during the discharge process is determined by the formula

$$\begin{aligned} T(z,t)/T_0^* = \frac{1}{2}\{&[\operatorname{erf}(\tilde{h}+\tilde{z}) + \operatorname{erf}(\tilde{h}-\tilde{z})] \\ &+ \frac{z}{h}[\operatorname{erf}(\tilde{h}+\tilde{z}) - \operatorname{erf}(\tilde{h}-\tilde{z}) - 2\operatorname{erf}\tilde{z}] \\ &+ \frac{1}{\sqrt{\pi}}\frac{1}{\tilde{h}}\{\exp[-(\tilde{h}^2+\tilde{z}^2)] \\ &+ \exp[-(\tilde{h}^2-\tilde{z}^2)] - 2\exp(-\tilde{z}^2)\}\}, \end{aligned} \tag{2}$$

where

$$\tilde{h} = \frac{H}{2\sqrt{Kt}}, \qquad \tilde{z} = \frac{z}{2\sqrt{Kt}}.$$

For the surface temperature we have

Fig. 4. Temperature variation record at a certain level in a layer of liquid when it is heated from below; one division along the t axis equals 10 sec; 2—schematic of the change in the mean temperature gradient in the layer

Figure 2: Fig. 4. Temperature variation record at a certain level in a layer of liquid when it is heated from below; one division along the t axis equals 10 sec; 2—schematic of the change in the mean temperature gradient in the layer

$$T(0,t) = T_0^* \{\operatorname{erf} \tilde{h} - (\sqrt{\pi}\tilde{h})^{-1}[1 - \exp(-\tilde{h}^2)]\}.$$

The curves characterizing the process of thermal discharge of the layer are shown in Fig. 3 (4, 5, 6). The change in the mean gradient in the layer is represented schematically by curve 2 in Fig. 4. The accumulation time t_{n} and the discharge time t_{p} are determined from the formulas for the surface temperature $T(0,t)$. The accumulation time is $t_{\text{n}} = \pi (T_0^*)^2 / 4\varphi_0^2 \chi$. If $\gamma^* = T_0^*/H$ is introduced into consideration, then the formula for t_{n} can be rewritten in the form $4\chi t_{\text{n}}/H^2 = \pi(\gamma^*)^2/\varphi_0^2$. The deviation of the surface temperature T_0^* relaxes in the course of discharge to a certain value T_1^*, which is approached asymptotically by the temperature curves at all levels in the H-layer. Therefore the discharge time t_{p} is determined from the relation $T(0,t_{\text{p}})'/T_0^* = T_1^*/T_0^* = \delta < 1$. For large–

Fig. 4. 1 —record of the course of temperature at a certain level in a layer of liquid when it is heated from below; one division along the t axis is equal to 10 sec; 2 —schematic of the change in the mean temperature gradient in the layer

values of Kt, the desired ratio of the characteristic times is

$$\frac{t_{\text{n}}}{t_{\text{p}}} \cong \left[\frac{\pi}{\sqrt{\pi} - 1}\right]^2 \frac{(\gamma^*)^2 \delta^2}{\varphi_0^2} \frac{K}{\chi}.$$

That is, the character of the process is determined by the ratio of the thermal-diffusivity coefficients K/χ in the periods of accumulation and discharge. For large values of K/χ, the discontinuity of the process is pronounced: short, violent discharge periods are separated by long intervals of accumulation.

The atmospheric layer with convective blocking considered here is an auto-oscillatory system which periodically draws definite portions of energy from a constant source of energy, i.e., by means of a nonperiodic source creates a periodic process. Temperature oscillations arise as the result of the action of negative feedback between temperature stratification and convective currents in the atmospheric layer.

As an illustration of temperature auto-oscillations, we shall present records obtained in a laboratory model. Figure 4 gives a curve of the course of temperature at a level of ~ 0.7 cm from the bottom of a vessel with water heated from below. The bottom of the vessel was metallic, and the walls were made of plexiglass. A sensitive low-inertia thermistor served as the sensor; recording was carried out with an electronic automatic self-recording poten-

of the EPP-09 m^3 radiometer. A characteristic feature of the curve is the regularity of the temperature bursts, the alternation of prolonged periods of temperature rise and short periods of its decline.

The processes of formation and decay of convective tiers in the atmosphere are, of course, much more complex than the scheme described. In the atmosphere there is not a single tier, but a system of interacting tiers. The formation of the overlying tier occurs through the swelling of the lower one. In the process of discharge of a layer, the effective thermal conductivity is a function of z and t. It is necessary, of course, to take account of wind and turbulence. However, the scheme makes it possible to consider from a new point of view the processes of heat and mass exchange in atmospheric layers. These processes may be intermittent and oscillatory. At the same time, it becomes possible to understand better certain phenomena in the atmosphere that are paradoxical from the "stationary" point of view. Thus, the question of an "equilibrium" gradient[6] is debatable. From the nonstationary oscillatory point of view it is more convenient to discuss this question. Indeed, since oscillations of the gradient occur, measurements give mean values of the gradient, which are below the critical one. But when determining the conditions of convective instability one must use not the mean, but the true value of the critical gradient. Let us also note that the alternation in time of processes of accumulation and discharge makes the convective system self-maintaining. The expenditure of energy in it during the period of convective currents is restored during the subsequent refractory period (period of rest).

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Note: Figure translations are in progress. See original paper for figures.