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Abstract

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Turbulent mixing of a heavy fluid leads to mutual transformations of the kinetic energy of turbulence and the potential energy of a stratified fluid, occurring at the rate

$$\frac{\partial K}{\partial t} = -\frac{g}{\rho} M; \qquad M = \overline{\rho' w'}, \tag{1}$$

where K is the kinetic energy of turbulence per unit mass; t is time; g is the acceleration of gravity; ρ is the density of the fluid; ρ' and w' are turbulent fluctuations of density and vertical velocity; M is the vertical turbulent mass flux; the overbar denotes averaging. Under stable stratification, when density increases downward, the flux M is directed upward (positive), and mixing leads to expenditure of kinetic energy in overcoming Archimedean forces. Conversely, under unstable stratification, when density increases upward, the flux M is directed downward (negative), and the Archimedean forces, generating convective turbulence, contribute to the generation of kinetic energy.

Thus, the distribution of M along the vertical is a dynamical characteristic of the stratification of a fluid from the standpoint of turbulence. Changes of M along the vertical (divergence of the mass flux) may lead to a change with time of the density stratification, and in a statistically stationary and horizontally homogeneous regime (and in the presence of a solid horizontal wall—the bottom) only to the appearance of a compensating vertical velocity $\overline{w} = -M/\rho$.

In the atmosphere $\rho' \approx -\rho T'/T$, where T is temperature, and $M = -q/c_{pT}$, where $q = c_p \rho \overline{T' w'}$ is the vertical turbulent heat flux, c_p is the specific heat of air at constant pressure. Measurements in the atmosphere give for q values of the order of 10^{-2} cal/cm$^2 \cdot$ min under moderately stable stratification and 10^{-1} under instability. With $c_p = 0.24$ cal/g $\cdot$ deg and $T \sim 300°$, these correspond to values of M of the order of 10^{-6} and 10^{-5} g/cm$^2 \cdot$ sec (or 10^2 and 10^3 g/cm$^2 \cdot$ yr).

In the ocean neither q nor M has yet been measured. Since for water the coefficient of thermal expansion $\alpha \sim 2 \cdot 10^{-4}$ deg^{-1} is an order of magnitude smaller, while the density is three orders of magnitude larger, than for air, for the same temperature fluctuations the density fluctuations produced by them in water should be two orders of magnitude greater than in air. But information on the orders of magnitude of the fluctuations T' and w' in the ocean is still quite insufficient. Therefore, to estimate the values of M in the ocean one must for the time being resort to indirect methods. One of them may be the use of the boundary condition for M at the ocean surface

$$M_0 = (P - E)S - \frac{\alpha}{c}(\mathcal{L}E + Q) + \alpha P(T_p - T_w). \tag{2}$$

The first term here describes the change in the salinity S of the surface layer of water during the fall of precipitation P (freshening) and during evapora-

E (salinization). For mean values of P and E of the order of 1 m of water per year and for $S \sim 3 \cdot 10^{-2}$, this term is of the order of 10^{-7} g/cm$^2 \cdot$ sec, but during periods of intense precipitation or evaporation it may quite well be one or even two orders of magnitude larger. The second term describes thermal changes in the density of the surface layer of water due to heat exchange with the atmosphere (c is the heat capacity of water, $\mathcal{L} \approx 600$ cal/g is the latent heat of evaporation; Q is the sum of the turbulent and radiative heat fluxes in the air at the water surface, positive when the flux Q is directed upward). Since $\alpha\mathcal{L}/c \approx 0.12$, the contribution of evaporation here is four times greater than in the first term; the contribution of Q is of the same order. Finally, the third term describes thermal changes in the density of water due to the heat or cold brought by precipitation (T_p and T_w are the temperatures of the precipitation and of the surface layer of water). For $T_p - T_w \sim 10°$ this term is an order of magnitude smaller than the contribution of P in the first term.

The values of M in the internal layers of the ocean can be estimated with the aid of the "law of distance from the bottom" $N = A(H - Z)$ for the Brunt-Väisälä frequency

$$N = \left(\frac{g}{\rho}\frac{\partial \rho_{\mathrm{p}}}{\partial z}\right)^{1/2},$$

established in the work of A. S. Monin, V. G. Neiman, and B. N. Filyushkin ([1]) and valid at depths Z greater than 1.5-2 km. Here H is the total depth of the ocean; ρ_{p} is the potential density; A is a constant whose values in different regions of the ocean vary within the limits 10^{-7}-10^{-6} m$^{-1} \cdot$ sec^{-1}. Using similarity theory for turbulence in a stably stratified fluid, the constant A can be interpreted by the formula $A = \Gamma/L$, where L and Γ are the typical scale of the largest turbulent inhomogeneities and the gradient of the mean current velocity at distances from the bottom large in comparison with L, determined by the formulas

$$L = u_*^3 \left(\varkappa \frac{g}{\rho} M \right)^{-1}; \qquad \Gamma = \frac{u_*}{\varkappa R L}, \tag{3}$$

where $u_* = (\tau/\rho)^{1/2}$ is the friction velocity (τ is the vertical turbulent flux of momentum); $\varkappa \approx 0.4$ is the von Kármán constant; $R \approx 0.1$ is the limiting (maximum) value of the dynamic Richardson number Rf. From the interpretation of A and formulas (3), for M one obtains the expression

$$M = \frac{\rho}{g} u_*^{5/2} \left(\frac{RA}{\varkappa} \right)^{1/2}. \tag{4}$$

If we accept that a significant fraction of the momentum flux ρu_*^2 transmitted from the air to the water goes to the drift current, then the ratio of the values of u_* in water and in air will be of the order of the square root of the ratio of the densities of air and water, equal to 0.03. Since in air u_* is of the order of tens of centimeters per second, in water $u_* \sim 1$ cm/sec. Using this estimate and the empirical values of A indicated above, from (4) we obtain $M \sim 10^{-8}$ g/cm$^2 \cdot$ sec (or only ~ 1 g/cm$^2 \cdot$ year), i.e., a value two orders of magnitude smaller than under stable stratification in the atmosphere. The values of L corresponding to the estimates given for u_* and M are of the order of a kilometer. From this point of view, the internal layers of the ocean may be characterized as a medium with weakly stable stratification (whereas very stable stratification occurs only in the so-called jump layer).

The smallness of M in the internal layers of the ocean can be explained by the fact that turbulence there has an intermittent character—it is concentrated only in thin, pancake-like patches (formed, apparently, mainly as a result of the loss of stability of internal waves). Introducing the relative area χ of turbulent patches at a given depth (the intermittency coefficient), the formula for M can be written in the form $M = \chi \langle \rho' w' \rangle$, where the angle brackets denote averaging over turbulent patches. According to the measurements of Grant, Moilliet, and Vogel ([2]), even at

at small depths $Z = 100$–300 m the mixing coefficient χ is of order 10^{-2}, i.e., precisely of the order of the ratio of the values of M in the interior layers of the ocean and in the atmosphere. Then the values $\langle \rho' w' \rangle$ in the ocean and $\rho' w'$ in the atmosphere will be comparable.

Empirical data show that the constant A in the "law of distance from the bottom" proves to be the smaller, the greater the total depth of the ocean H. According to (4), the same statement is also valid for the values of M in the interior layers of the ocean, i.e., the thicker the ocean, the smaller the turbulent mass flux that it "lets through." To obtain a quantitative formulation, let us use the empirical "law of distance from the surface" established in [1], $NZ = w_*$, valid in the depth interval $Z = 500$-5000 m, where w_* is a constant varying little from one ocean region to another and having the value $w_* \approx 2.2$ m/sec. Equating, in the middle of the ocean $Z = H/2$, the expressions for N according to both laws, we obtain $w_* = A(H/2)^2$. Using the decoding of the constant A, the latter equality may be reduced to the form

$$\frac{L}{H} = \left(\frac{1}{4\chi R}\,\frac{u_*}{w_*}\right)^{1/2} = \delta. \tag{5}$$

The quantity δ is of order 10^{-1}. Hence we finally obtain

$$M = \frac{\rho}{g}\,\frac{u_*^3}{\chi\delta H}, \tag{6}$$

and if M is measured in g/cm^2·sec and H in km, then the product MH proves to be a quantity of order 10^{-7}, independent of H.

Let us now make use of the obvious relations

$$u_*^2 \sim \chi r_{uw} w_t^2; \qquad M \sim \chi r_{\rho w} w_t \rho_t, \tag{7}$$

where the subscript t denotes typical scales of fluctuations in turbulent patches, and r_{uw} and $r_{\rho w}$ are the correlation coefficients between the corresponding fluctuations. Then from (6) the following estimate is obtained for the density fluctuations in turbulent patches:

$$\frac{\rho_t'}{\rho} \sim \frac{1}{\chi\delta r_{\rho w}}\left(\frac{r_{uw}}{\chi}\right)^{1/2}\frac{u_*^2}{gH}. \tag{8}$$

This quantity is of order 10^{-6}, i.e., the fluctuations ρ_t' may be of the same order or even an order of magnitude greater than density fluctuations in the atmosphere under stable stratification. From what has been said above it then follows that temperature fluctuations in turbulent patches in the ocean may be considerably smaller than in the atmosphere, while fluctuations w_t' may be comparable with atmospheric ones or prove to be an order of magnitude smaller.

A number of the numerical estimates given depend substantially on the adopted value of u_* (~ 1 cm/sec), but can without difficulty be recalculated for cases in which it will be reasonable to adopt smaller values for u_*.

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2. H. L. Grant, A. Moilliet, W. M. Vigel, *J. Fluid Mech.*, **34**, Part 3 (1968).

Note: Figure translations are in progress. See original paper for figures.