

STRUCTURAL REPRESENTATION OF CERTAIN CLASSES OF HARMONIC AND ANALYTIC FUNCTIONS

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Abstract

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MATHEMATICS

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STRUCTURAL REPRESENTATION OF CERTAIN CLASSES OF HARMONIC AND ANALYTIC FUNCTIONS

(Presented by Academician M. A. Lavrent'ev on 19 I 1970)

M. M. Dzhrbashyan ⁽¹⁾ gave a complete structural representation for the classes of harmonic and analytic functions associated with the operator $L^{(\omega)}$ constructed by him ⁽¹⁾. In the present note a complete structural representation is given for the classes of harmonic and analytic functions introduced here and associated with the operator $L^{(\omega_1, \dots, \omega_m)}$ ⁽²⁾.

Let $U_{(\omega_1, \dots, \omega_m)}$ denote the set of functions $u(z)$ harmonic in the disk $|z| < 1$ and satisfying the condition

$$\sup_{0 < r < 1} \left\{ \int_0^{2\pi} |u_{(\omega_1, \dots, \omega_m)}(re^{i\varphi})| d\varphi \right\} < +\infty,$$

where $\omega_j(x) \in \Omega$ ($j = 1, \dots, m$)* and

$$u_{(\omega_1, \dots, \omega_m)}(re^{i\varphi}) = L^{(\omega_1, \dots, \omega_m)}[u(re^{i\varphi})].$$

Let $C_{(\omega_1, \dots, \omega_m)}$ denote the class of functions $f(z)$ analytic in the disk $|z| < 1$ and satisfying the condition

$$\operatorname{Re} f_{(\omega_1, \dots, \omega_m)}(z) \geq 0 \quad (|z| < 1),$$

where $\omega_j(x) \in \Omega$ and

$$f_{(\omega_1, \dots, \omega_m)}(re^{i\varphi}) = L^{(\omega_1, \dots, \omega_m)}[f(re^{i\varphi})].$$

Let $R_{(\omega_1, \dots, \omega_m)}$ denote the class of functions $f(z)$ analytic in the disk $|z| < 1$ for which the condition

$$\sup_{0 < r < 1} \left\{ \int_0^{2\pi} |\operatorname{Re} f_{(\omega_1, \dots, \omega_m)}(re^{i\varphi})| d\varphi \right\} < +\infty.$$

is satisfied. Since, for $f(z) \in C_{(\omega_1, \dots, \omega_m)}$, the integrals

$$\int_0^{2\pi} |\operatorname{Re} f_{(\omega_1, \dots, \omega_m)}(re^{i\varphi})| d\varphi = \int_0^{2\pi} \operatorname{Re} f_{(\omega_1, \dots, \omega_m)}(re^{i\varphi}) d\varphi = 2\pi \operatorname{Re} f_{(\omega_1, \dots, \omega_m)}(0)$$

are bounded by a number independent of $r < 1$, we obviously have

$$C_{(\omega_1, \dots, \omega_m)} \subset R_{(\omega_1, \dots, \omega_m)}.$$

Let $P(\theta, r; \omega_1, \dots, \omega_m)$, $S(z; \omega_1, \dots, \omega_m)$ be the functions introduced by the author in ⁽²⁾.

* It is said (see ⁽¹⁾, p. 1078) that a function $\omega(x) \in \Omega$ if it is nonnegative and continuous on $[0, 1)$, with

$$\omega(0) = 1, \quad \int_0^1 \omega(x) dx < +\infty$$

and, for every r ($0 \leq r < 1$),

$$\int_r^1 \omega(x) dx > 0.$$

Theorem 1. a) The class $U_{(\omega_1, \dots, \omega_m)}$ coincides with the set of functions $u(z)$ representable in the form of the integral

$$u(re^{i\varphi}) = \frac{1}{2\pi} \int_0^{2\pi} P(\varphi - \theta, r; \omega_1, \dots, \omega_m) d\psi(\theta) \quad (1)$$

$$(0 \leq r < 1, 0 \leq \varphi \leq 2\pi),$$

where $\psi(\theta)$ is an arbitrary real function with finite total variation on $[0, 2\pi]$.

b) In representation (1) of a given function

$$u(z) \in U_{(\omega_1, \dots, \omega_m)}$$

the corresponding function $\psi(\theta)$ can be determined by means of the limit

$$\psi(\theta) = \lim_{n \rightarrow +\infty} \int_0^\theta u_{(\omega_1, \dots, \omega_m)}(\rho_n e^{i\varphi}) d\varphi,$$

where $\{\rho_n\}$, $0 < \rho_1 < \rho_2 < \dots < \rho_n < \dots$, $\rho_n \uparrow 1$, is some increasing sequence.

c) The class

$$U_{(\omega_1, \dots, \omega_m)}^* \subset U_{(\omega_1, \dots, \omega_m)}$$

of functions harmonic in the disk $|z| < 1$, for which

$$u_{(\omega_1, \dots, \omega_m)}(z) \geq 0 \quad (|z| < 1),$$

coincides with the set of functions representable in the form (1), where the function $\psi(\theta)$ is nondecreasing on $[0, 2\pi]$.

Theorem 2. a) The class $C_{(\omega_1, \dots, \omega_m)}$ coincides with the set of functions $f(z)$ representable in the form

$$f(z) = iC + \frac{1}{2\pi} \int_0^{2\pi} S(e^{-i\theta} z; \omega_1, \dots, \omega_m) d\psi(\theta) \quad (2)$$

$$(|z| < 1),$$

where $\text{Im } C = 0$, and $\psi(\theta)$ is an arbitrary nondecreasing bounded function on $[0, 2\pi]$.

b) The class $R_{(\omega_1, \dots, \omega_m)}$ coincides with the set of functions representable in the form (2), where $\psi(\theta)$ is a real function of finite variation on $[0, 2\pi]$.

In the proof of Theorem 1, Theorem 2 of the author from ⁽²⁾ is used essentially, and in the proof of Theorem 2—the author's Theorem 1 from ⁽²⁾ and Theorem 1 of the present note.

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References

- ¹ M. M. Dzhrbashyan, *Izv. AN SSSR, Ser. Mat.*, **32**, No. 5, 1075 (1968).
- ² I. I. Bavrín, *DAN*, **187**, No. 3 (1969).

Note: Figure translations are in progress. See original paper for figures.

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