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Abstract

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CYBERNETICS AND CONTROL THEORY

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FINITE CONTROL AND CONTROLLABILITY IN DISTRIBUTED SYSTEMS

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An interesting problem of finite control (i.e., control on a finite time interval $[0, T]$) ([1,2]) is the problem of damping oscillations in a branched and interconnected network of one-dimensional oscillatory systems (in particular, an electric power network with several power stations and consumers connected to one another by long transmission lines), for example, the network shown in Fig. 1. Here C_1 and C_2 denote power stations, and P_1, P_2, P_3 are consumers.

Fig. 1 **Fig. 2**

Let us consider the simplest problem of this type (Fig. 2), in which there is only one control at the point C and two lines of lengths S_1 and S_2 with "fixed" ends P_1 and P_2. The precise formulation of such a problem in this simplest case is as follows. Suppose that the oscillations in the lines S_1 and S_2 are described respectively by the functions

$$Q_1(x_1, t), \qquad 0 \le x_1 \le S_1,$$

$$Q_2(x_2, t), \qquad 0 \le x_2 \le S_2,$$

for $t \ge 0$.

The equations for these functions have the form:

$$\ddot{Q}_1 = Q_1'', \qquad 0 < x_1 < S_1, \qquad t > 0, \tag{1}$$

$$\ddot{Q}_2 = Q_2'', \qquad 0 < x_2 < S_2, \qquad t > 0, \tag{2}$$

where dots denote partial derivatives with respect to time t, and primes denote partial derivatives with respect to the spatial variables x_1 and x_2.

The boundary and initial conditions have the form:

$$Q_1(0,t) = Q_2(0,t) = u(t), \tag{3}$$

$$Q_1(S_1,t) = Q_2(S_2,t) = 0, \tag{4}$$

$$Q_1(x_1,0) = Q_{01}(x_1), \qquad \dot{Q}_1(x_1,0) = Q_{11}(x_1), \tag{5}$$

$$Q_2(x_2,0) = Q_{02}(x_2), \qquad \dot{Q}_2(x_2,0) = Q_{12}(x_2), \tag{6}$$

$$0 \le x_1 \le S_1, \qquad 0 \le x_2 \le S_2, \qquad t \ge 0.$$

The problem of finite control is to bring these systems to rest by means of the control $u(t)$, $0 \le t \le T$, i.e., to obtain the conditions

$$Q_1(x_1,T) = \dot{Q}_1(x_1,T) = 0, \qquad 0 \le x_1 \le S_1, \tag{7}$$

$$Q_2(x_2,T) = \dot{Q}_2(x_2,T) = 0, \qquad 0 \le x_2 \le S_2. \tag{8}$$

The first essential question that arises in solving this problem is: is this system controllable? In other words: does there exist a finite control $u(t)$, $0 \le t \le T$, for finite T and arbitrary initial conditions (5), (6)? It turns out that, by the method of finite control, this problem is solved very simply, and the answer is negative. Indeed, the interpolation problem for this problem, as follows from formulas (13) in [1], has the form

$$\tilde{u}(m\pi/S_1) = \beta_{1m}, \qquad \tilde{u}(n\pi/S_2) = \beta_{2n}, \qquad m,n = \pm 1, \pm 2, \dots, \tag{9}$$

where the numbers β_{1m} and β_{2n} are computed by formula (14) in paper [1] with the corresponding changes of variables, in which, to compute β_{1m}, one must put $Q_{01}(x_1)$ instead of $Q_0(x)$, $Q_{11}(x_1)$ instead of $Q_1(x)$, while to compute β_{2n} one must take $Q_{02}(x_2)$ instead of $Q_0(x)$, and $Q_{12}(x_2)$ instead of $Q_1(x)$. From (9) it is immediately clear that if the numbers S_1 and S_2 are commensurable, then for any $n = \pm 1, \pm 2, \dots$ there will be an m from the sequence $\pm 1, \pm 2, \dots$ such that

$$m/S_1 = n/S_2. \tag{10}$$

Since, by virtue of the arbitrariness of the initial distributions $Q_{01}(x_1)$, $Q_{11}(x_1)$, $Q_{02}(x_2)$, and $Q_{12}(x_2)$, the numbers β_{1m} and β_{2n}, $m, n = \pm 1, \pm 2, ...$, are different, the interpolation problem (9), in the case where S_1 and S_2 are commensurable, is unsolvable in entire functions $\tilde{u}(z)$ of given finite degree, since the equality

$$\tilde{u}(m\pi/S_1) = \tilde{u}(n\pi/S_2)$$

must hold, whereas, generally speaking, at the same time $\beta_{1m} \neq \beta_{2n}$.

It can also be shown that this interpolation problem (9) is unsolvable in the case of incommensurable S_1 and S_2. This can easily be shown by contradiction, using the fact that an entire function of finite degree belonging on the real axis to $L_2(-\infty, \infty)$ has a bounded derivative [3].

Similarly, it can be shown that the system will remain uncontrollable if a new control $v(t)$ is added at the right-hand ends of the intervals $[0, S_1]$ and $[0, S_2]$, i.e., if condition (4) is replaced by the condition

$$Q_1(S_1, t) = Q_2(S_2, t) = v(t), \qquad t \geq 0. \tag{11}$$

In this case the interpolation problem has the form

$$\tilde{u}(m\pi/S_1) - (-1)^m \tilde{v}(m\pi/S_1) = \beta_{1m}, \tag{12}$$

$$\tilde{u}(n\pi/S_2) - (-1)^n \tilde{v}(n\pi/S_2) = \beta_{2n}, \tag{13}$$

$$m, n = \pm 1, \pm 2, \dots, n,$$

whence it is seen that, in the case of commensurable S_1 and S_2, one can find such m and n that the left-hand sides of equations (12) and (13) will be identical, whereas the right-hand sides β_{1m} and β_{2n}, generally speaking, are different. The unsolvability of the interpolation problem (12) and (13) can also be shown in the case of incommensurable S_1 and S_2.

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Note: Figure translations are in progress. See original paper for figures.