On the Influence of an Admixture of Solid Particles or Droplets on the Structure of a Turbulent Gas Jet

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G. N. Abramovich

On the Influence of an Admixture of Solid Particles or Droplets on the Structure of a Turbulent Gas Jet

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The problem of the propagation of a nonuniform turbulent jet is of great practical interest and has recently attracted the attention of a number of researchers both in the USSR and in other countries. The most difficult question is that of the influence on a gas jet of a discrete admixture of solid particles or liquid droplets.

Let us introduce the concept of the mean density of the substance of the jet at a given point:

$$\rho = \rho_{\mathrm{g}}(1+\chi), \tag{1}$$

where ρ_{g} is the gas density, $\chi = G_{\mathrm{p}}/G_{\mathrm{g}}$ is the weight concentration of the admixture.

Experience shows that, for the same mean density, a jet consisting of a mixture of two gases of different molecular weight behaves differently from one containing an admixture in the form of discrete particles. In the latter case the jet proves to be narrower and to have a greater range. A probable explanation of this fact is that the turbulent structure of the jet is modified under the influence of the heavy particles. We shall show that, in a first approximation, the action of solid particles on the pulsation velocities and averaged parameters of a turbulent flow can be estimated on the basis of the fundamental propositions of Prandtl's mixing-length theory.

Usually the size D of the solid particles or droplets carried by a gas jet is several orders of magnitude smaller than the size l of a turbulent eddy, whose behavior determines the structure of the turbulent flow. In the process of turbulent pulsating motion, a discrete gas volume (eddy) entrains the foreign particles inhabiting it and is retarded by the total force of the drag of the latter, as a result of which the pulsation components of the velocity of the turbulent flow decrease.

Let us consider the practically important case in which the weight of admixture per unit volume is significant, while the volume fraction of the admixture is relatively small. In this case the aerodynamic resistance of all the heavy particles, which are flowed around by the gas particles, is relatively large, but is equal to the sum of the resistances arising when each heavy particle is flowed around as an isolated body.

Let us examine the action of heavy particles on the magnitude of the transverse pulsation velocity of a turbulent jet $|v'|$, which, as is known ([1]), determines the intensity of growth of the jet thickness.

When a turbulent eddy arises, its pulsation velocity is proportional to the transverse gradient of the mean velocity of the flow,

$$|v_0'| \sim l_\text{T}\, du/dy; \tag{2}$$

here l_T is the mean value of the turbulent mixing length.

In semiempirical theories of turbulence it is assumed that an eddy retains its pulsation velocity throughout the entire time of its "life" —from the moment it separates from one layer of the flow until it merges with another layer; the loss of individuality of the eddy occurs abruptly, which leads to a pulsation of velocity (as well as of pressure, temperature, concentration, etc.).

From the condition of conservation of momentum for an isolated mole populated by the same impurity particles, it follows that, in the presence of foreign heavy particles, the velocity of the gas mole v' decreases in proportion to the increase in the velocity of motion of the particles v_p entrained by the mole, and the proportionality coefficient is equal to the ratio of the masses of impurity and gas, i.e., to the mass concentration of the impurity:

$$v_0' - v' = \varkappa(v_\text{p} - v_{\text{p}0}); \tag{3}$$

here $v_{\text{p}0}$ is the initial velocity of motion of the heavy particles. This relation is valid for a monodisperse impurity homogeneous in composition. In the case of a polydisperse impurity, the right-hand side of (3) becomes more complicated, but we shall return to this case later.

At the moment of formation of a turbulent gas mole, which begins to move with velocity v_0', heavy particles enter it which were brought by the preceding mole and, with equal probability, have velocity $v_{\text{p}0}$ directed either in the same or in the opposite direction. Therefore the mean value of the initial velocity of the heavy particles may be taken equal to zero ($v_{\text{p}0} \approx 0$), which simplifies equation (3):

$$v_0' - v' = \varkappa v_\text{p}. \tag{4}$$

In the case of a high mass concentration of impurity ($\varkappa \gg 1$), as follows from (4), even with considerable slowing of the mole ($v_0' \gg v'$), the heavy particles acquire a relatively small velocity ($v_\mathrm{p} \ll v_0'$). The further solution of the problem depends on how rapidly the terminal values of the velocities of the mole and of the heavy particles approach one another.

If the particle sizes and velocities are small, then the "intramolecular" flow of gas around the particles may be laminar; in that case the aerodynamic drag coefficients are relatively high and, owing to the relatively small mass of each particle, its velocity rapidly becomes equal to the gas velocity ($v_{\mathrm{p}k} \approx v_k'$), in connection with which, according to (4), the terminal velocity of motion of the gas mole is

$$v_k' \approx v_0' \frac{1}{1+\varkappa}. \tag{5}$$

It follows from this that the ratio of the turbulent shear stress in a flow with impurity to that in a pure gas (for $v' \sim u'$) is:

$$\frac{\tau}{\tau_0} = \frac{\rho u_k' v_k'}{\rho_\mathrm{g} u_0' v_0'} = \frac{1}{1+\varkappa}.$$

The intensity of increase of the jet thickness along its length, $d\delta/dx$, as is known, is proportional to the transverse pulsation velocity (1). In the case under consideration:

$$\frac{d\delta}{dx} \sim \frac{v_k'}{u_c} \quad \text{or} \quad \frac{d\delta}{dx} \sim \frac{v_k'}{v_0'}\frac{v_0'}{u_m}\frac{u_m}{u_c}, \tag{6}$$

here u_m is the velocity on the jet axis, u_c is the characteristic value of the mean flow velocity; the subscript 0 corresponds to the case of absence of impurity, when in the flooded jet

$$v_0' \sim u_m, \qquad v_0'/u_{c0} \sim (d\delta/dx)_0 = c = 0.22.$$

In a jet of variable density (with impurity), one may, using recommendation (1), set

$$u_c = \int_0^\delta \rho u \, dy \Big/ \int_0^\delta \rho \, dy \tag{7}$$

Numerous experiments show that in a jet containing both gaseous and droplet-liquid or dispersed solid impurities, the relative velocity profile in the cross section remains unchanged and, in the main portion of the jet, can be described by Schlichting' s formula

Fig. 1

Figure 1: Fig. 1

$$u/u_m = \left[1 - (y/\delta_u)^{1.5}\right]^2, \tag{8}$$

where u is the velocity at a distance y from the jet axis, and δ_u is the distance from the axis to the dynamic boundary of the jet (the jet radius).

The concentration profiles are satisfactorily described by the same dependence, but the corresponding thickness of the "impurity zone" usually differs from the dynamic one ($\delta_\chi \neq \delta_u$):

$$\chi/\chi_m = \left[1 - \eta^{1.5}\eta^{-1.5}\right]^2; \tag{9}$$

here $\eta = y/\delta_u$, $\eta = \delta_\chi/\delta_u$, $\eta\eta^{-1} = y/\delta_\chi$, and χ_m is the concentration on the jet axis. According to some experimental data, the dependence $\eta\,(\chi_m)$ can be expressed by the curve shown in Fig. 1*. As can be seen, in the interval $0.5 \leq \chi_m \leq 10$ the quantity $\eta < 1$, i.e., the velocity profile is broader than the concentration profile. In the absence of impurity ($\chi = 0$), from (7) and (8) we obtain $u_{c0} = 0.45u_m$.

Fig. 1

In the presence of a finely dispersed impurity ($v \approx v'_k$)

$$\frac{d\delta}{dx} = \frac{0.45a}{1+\chi_1}\,\frac{u_m}{u_c}; \tag{10}$$

here χ_1 is a certain characteristic value of the concentration in the cross section of the jet.

To determine the local value of the angular coefficient of growth of the jet thickness using (10), it is necessary to find a way of choosing the corresponding value of the characteristic concentration in the section, χ_1. It appears possible to assume that the dominant zone in the generation of turbulence in the jet section is the one where the turbulent pressure pulsations are maximal:

$$P'_{\max} = \rho v'^2_{\max} \sim \left[(1+\chi)(du/dy)^2\right].$$

Solving the maximum problem, we find the coordinate of this point η_1. Calculations show that the quantity χ_1/χ_m in the interval $0.5 \leq \chi_m \leq 10$ changes only slightly: $\chi_1/\chi_m = 0.76 \pm 0.03$. The curve calculated by this method,

$$\frac{1}{C}\frac{d\delta}{dx} = \psi(\chi_m),$$

is shown in Fig. 1. Recall that ψ is the ratio of the intensity of growth of the thickness of a jet with impurities to that in a "pure" jet.

The results obtained are valid only for a relatively finely dispersed impurity, which is fully entrained by the turbulent eddies of the gas. Approximate calculations show that, at an average velocity in the jet of about 50 m/sec, solid particles with diameters up to 30 μ are fully entrained. The situation is more complicated with relatively large particles, which do not have time, during the motion of an eddy, to acquire its velocity, $(v < v'_k)$.

* The solid curve $\eta\,(\chi_m)$ in Fig. 1 was obtained from experiments on the mixing of two gases of different molecular weight (freon and air) and is probably valid in the case of a very finely dispersed impurity ($D < 30\ \mu$). The experimental data of Frishman and Laats in the case of relatively large impurity particles ($D \approx 30 \div 120\ \mu$) correspond to the dashed curve in Fig. 1.

Let us introduce the relative velocity $v_\sim = v' - v_\mathrm{p}$. From (4) we obtain

$$\frac{v'_k}{v'_0} = \frac{1 + \chi v_{\sim k}/v'_0}{1+\chi}. \tag{11}$$

Thus, in the case where the solid particles lag behind the gas moles ($v_{\sim k} = v'_k - v_{\mathrm{p}k} > 0$), formula (11) is substituted into (6) instead of (5); the intensity of increase of the jet thickness is then greater than in the case considered above ($v_{\sim k} = 0$):

$$\frac{d\delta}{dx} = 0.45C\frac{u_m}{u_c}\frac{1+\chi_1 v_{\sim k}/v'_0}{1+\chi_1}. \tag{12}$$

The relation between the current values of the velocity of the gas mole v' and of the solid (spherical) particle is established by the equation of Newton' s second law of mechanics:

$$m\frac{dv_\mathrm{p}}{dt} = C_x\frac{\pi D^2}{4}\frac{\rho_\mathrm{g}}{2}(v' - v_\mathrm{p})^2,$$

where the particle mass is $m = {}^4/_3\pi\rho_\mathrm{p}(D/2)^3$. The distance traveled by the particle during the time dt is equal to $dy = v_\mathrm{p}dt$, therefore

$$\frac{v_\mathrm{p}\,dv_\mathrm{p}}{(v' - v_\mathrm{p})^2} = \frac{0.75C_x}{D}\frac{\rho_\mathrm{g}}{\rho_\mathrm{p}}\,dy. \tag{13}$$

An estimate of the effect of unsteadiness of the flow around the particle on the value of the drag coefficient, made from the data of work [2], showed that this effect does not exceed 10%; therefore one may use the usual dependences

$C_x(R_D)$ for steady flow, determining the Reynolds number from the relative velocity of the pulsational motion $v_\sim$.

The value of the dimensionless relative velocity at the end of the unit displacement of the mole, $v_{\sim k}/v'_0$, is determined from (13) at $y = l_\text{T}$. The dependences obtained by us, which characterize a jet with relatively large particles, are valid only for a monodisperse impurity.

In the case of a polydisperse impurity the relation

$$\chi = \chi_a + \chi_b + \dots, \tag{14}$$

is valid, where $\chi_a, \chi_b, \dots$ are the mass concentrations of particles with diameters $D_a, D_b, \dots$. Let the velocities to which particles of different sizes are accelerated be $v_{\text{p}a}, v_{\text{p}b}, \dots$. Then equation (4) for a polydisperse mixture has the form

$$v'_0 - v' = \chi_a v_{\text{p}a} + \chi_b v_{\text{p}b} + \dots \tag{15}$$

The subsequent calculation is based on the simultaneous solution of the system of equations (13), written for particles of each size, and equation (15). As a result, at $y = l_\text{T}$ one can determine the quantities $v_{\text{p}a}, v_{\text{p}b}, \dots$ and the value v'_k, which is substituted into (6) and gives the value of $d\delta/dx$ for the polydisperse mixture. The remaining jet parameters (the fields of concentration and velocity, the variation of the jet thickness along its length, etc.) are found with the aid of the conservation laws for the total values of momentum and impurity mass.

The calculations become somewhat more complicated when the inequality of the local time-averaged velocities of the gas and of the solid particles is taken into account. If these velocities differ, then, because of the rotation of the solid particles (owing to the vorticity of the flow), an aerodynamic lift force arises, which displaces the particles in a direction perpendicular to the jet axis; this affects the character of the dependence $\eta_\text{p}(\chi_m)$. The corresponding curve in Fig. 1 does not take the influence of this factor into account.

The available experimental data do not contradict the simplified theory described; for a more rigorous assessment of it, careful and detailed experimental investigations must be carried out.

Moscow Aviation Institute
named after S. Ordzhonikidze

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REFERENCES

[1] G. N. Abramovich, *Theory of Turbulent Jets*, Moscow, 1960. [2] A. S. Ginevskii, K. K. Fedyaevskii, *Izv. AN SSSR, OTN, Mechanics and Mechanical Engineer-*

ing, No. 3 (1959).

Note: Figure translations are in progress. See original paper for figures.