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AN EFFECTIVE METHOD FOR SOLVING THE RADIATION TRANSPORT EQUATION IN A LOW-TEMPERATURE PLASMA

(Presented by Academician A. N. Tikhonov on 8 IV 1970)

1. Radiation transport in a plasma is described by the equation ([1])

$$\Omega\nabla I_\nu + \chi_\nu I_\nu = \chi_\nu I^*_{\nu p}. \tag{1}$$

The gasdynamic quantities (T, ρ, etc.) are functions of $\mathbf{r}$ and t. The radiation intensity $I_\nu(\mathbf{r}, \Omega, \nu, t)$ depends on a much larger number of variables. In our papers ([2],[3]) a method is described for effectively taking into account the dependence on the angular variables. The use of averages of the absorption coefficient according to Planck, Rosseland ([1]), or their combinations ([4],[5]), makes it possible to replace equation (1) by a one-group equation. However, the use of such averages is justified only for an optically thin body or when the radiation is close to equilibrium. In the remaining cases, the complicated nonmonotonic dependence of the absorption coefficient χ'_ν on frequency makes it necessary to use a large number of groups (a fine grid in ν).

For problems in which the physical quantities do not depend on one Cartesian coordinate, a new averaged equation is constructed. It is based on the exact solution obtained when a) $\chi'_\nu(\nu, T, \rho) = f_1(\nu) \cdot f_2(T, \rho)$. This averaging is naturally extended to the case b) $\chi'_\nu = f_1(\nu, T, \rho) \cdot f_2(T, \rho)$, where f_1 is a slowly varying function over distances of the order of the mean free path. This makes it possible to use the proposed method in calculations of real problems when the absorption coefficient contains a factor accounting for re-emission $(1 - e^{-h\nu/kT})$. Apart from requirements a) and b), no other requirements are imposed on f_1 and f_2, nor on the optical thickness of the body or on the magnitude of the gradients of T, ρ. Using the specific physical properties of the problem, the averaging obtained can be applied when f_1 varies strongly over distances of the order of the mean free path.

2. For a plane layer in case a), equation (1) is replaced by the averaged equation

$$z\frac{dI}{dx} = f_2(F(|z|,T) - I),$$

$$-\max_\nu f_1^{-1}(\nu) \le z \le \max_\nu f_1^{-1}(\nu), \qquad (2)$$

$$F(|z|,T) = 2\pi \int_{f_1^{-1}(\nu)\ge|z|} I_{\nu p}\cdot f_1^2(\nu)\,d\nu,$$

where the integration extends over those frequencies for which $f_1^{-1}(\nu) \ge |z|$. (The method of integration corresponds to the definition of the Lebesgue integral.) For equation (2) the boundary conditions are used (for example, for the left end)

$$I(0,z) = 2\pi \int_{f_1^{-1}(\nu)\ge z} I^+(0, z\cdot f_1(\nu),\nu)\cdot f_1^2(\nu)\,d\nu, \qquad (3)$$

* The notation corresponds to that in paper (^{1}).

where $I^+(0,\mu,\nu)$ is the boundary condition for equation (1). The equality holds

$$\int_{-\max_\nu f_1^{-1}(\nu)}^{\max_\nu f_1^{-1}(\nu)} zI\,dz = \int_0^\infty d\nu \int_{-1}^1 \mu I_\nu^*\,d\mu,$$

which makes it possible to find the radiant energy flux from the solution of equation (2). In an analogous manner one can write equations for determining other integral quantities.

For two-dimensional geometry the averaged equation takes the form

$$\gamma\frac{\partial I}{\partial s} = f_2(F(\gamma,T) - I), \qquad 0 \le \gamma \le \max_\nu f_1^{-1}(\nu), \qquad (4)$$

where $\partial/\partial s$ is the derivative in the direction s in the x,y plane,

$$F(\gamma,T) = 2\int_{f_1^{-1}(\nu)\ge\gamma} \frac{I_{\nu p}\, f_1^2(\nu)\,d\nu}{\sqrt{f_1^{-2}(\nu) - \gamma^2}}$$

with boundary condition

$$I(\mathbf{r},\mathbf{s},\gamma) = 2\int_{f_1^{-1}(\nu)\ge\gamma} \frac{I^+(\mathbf{r},\mathbf{s},\gamma\cdot f_1(\nu),\nu)\, f_1^2(\nu)\,d\nu}{\sqrt{f_1^{-2}(\nu) - \gamma^2}} \qquad (5)$$

Fig. 1 and Fig. 2

Figure 1: Fig. 1 and Fig. 2

for **r** on the boundary of the body, **s** directed inward into the body.

Instead of tables of $\varkappa'_\nu(\nu, T, \rho)$, in computing equations (2) and (4) it is necessary to use tables $F(|z|, T)$ and $F(\gamma, T)$. We note that F is a monotonically decreasing function of γ or $|z|$. This makes it possible, even with a very complicated spectral dependence $\varkappa'_\nu$, to use coarse grids in z and γ. Various efficient methods may be used to solve equations (2) and (4). In our calculations a quasi-diffusion method was used ([2];[3]).

3. In case b), equations (2) and (4) retain their form. Correspondingly, in this case

$$|z| \le \max_{x,\nu} f_1^{-1}(\nu, \rho, T), \qquad \gamma \le \max_{x,y,\nu} f_1^{-1}(\nu, \rho, T),$$

$$F(|z|, T, \rho) = 2\pi \int_{f_1^{-1}(\nu,T,\rho) \ge |z|} I_{\nu p}\, f_1^2(\nu, T, \rho)\, d\nu, \tag{6}$$

$$F(\gamma, T, \rho) = 2 \int_{f_1^{-1}(\nu,T,\rho) \ge \gamma} \frac{I_{\nu p} f_1^2(\nu, T, \rho)\, d\nu}{\sqrt{f_1^{-2} - \gamma^2}}. \tag{7}$$

In the case when $f_1(\nu, T, \rho)$ varies strongly over distances on the order of the mean free path, calculation of the flux by equations (2) and (4) gives the main error in the cold zones heated by radiation from hot zones. In a number of problems, in order to obtain the results of interest this phenomenon may be neglected and equations (2) and (4) may be used, despite the absence of separation of variables in $\varkappa'_\nu(\nu, T, \rho)$.

4. As an example, let us consider the problem of radiation emerging from a plane layer with temperature $T = 1$ for $0 \le x \le 0.015$, $T = 2$ for $0.015 \le x \le 0.03$, in the frequency interval $\nu \in (2; 4, 4)$. We specify the absorption coefficient by the formula $\varkappa'_\nu = f_1(\nu) \cdot T$, $f_1(\nu) = 100$ for $3 \le \nu \le 3, 2$, $f_1(\nu) = 1$ for the remaining ν. Using a very fine grid, we find the flux

W_T ("exact solution"). Using Planck and Rosseland averaging, we find, respectively, W_P and W_R. From the averaged equation given above we find W. A comparison of all the fluxes is given in Fig. 1.

As a second example, consider the problem of radiation incident on a plane layer of matter ([3,7,8]). Without taking phase transition into account, the process

Fig. 1 **Fig. 2**

is described by the equations of radiation gas dynamics. Radiation transport is described by (1). For a prescribed law of the incident radiation, the problem was calculated in the one-group T_1 and five-group approximation T_2 (^{3}) (Fig. 2). Recalculation of the problem by means of the averaged equation (2), T_3 (the absorption coefficient corresponding to the five-group approximation was regarded as exact), reproduced the results of the five-group calculation with good accuracy. In this case the separation (a) was violated by the reradiation factor.

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