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ON A PRINCIPLE UNDERLYING CLASSICAL PHYSICS

PHYSICS

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Abstract

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PHYSICS

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ON A PRINCIPLE UNDERLYING CLASSICAL PHYSICS

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A common feature of all physical laws is that the various physical objects belonging to certain classes are equivalent with respect to the law under consideration. Below a mathematical apparatus is set forth that makes it possible to formulate this concept of equivalence in a natural way. The general principle underlying the formulation of physical laws is written as a system of functional equations of a very special form. In subsequent papers we intend to show how this principle can be applied to questions of the foundations of known physical theories. This approach seems useful to us in that, with its aid, one can draw conclusions about the structural features of theories that have not yet been constructed but are possible in principle.

Consider two sets of arbitrary nature: a set $\mathfrak{M}$ with elements $i, k, \dots, l, \dots$ and a set $\mathfrak{N}$ with elements $\alpha, \beta, \gamma, \dots$. Suppose that to each pair $(i, \alpha) \in \mathfrak{M} \times \mathfrak{N}$ there corresponds a real number $a_{i\alpha} \in R$, so that on the set $\mathfrak{M} \times \mathfrak{N}$ a function $A : (i, \alpha) \to a_{i\alpha}$ is defined. (If $\mathfrak{M}$ and $\mathfrak{N}$ are sets of physical objects, then $a_{i\alpha}$ is the result of an experimental operation characterizing the relation in which the objects i and α stand.) Let m and n be natural numbers, and let $\mathfrak{M}_m = \{i_1, i_2, \dots, i_m\}$ and $\mathfrak{N}_n = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ be finite subsets of the sets $\mathfrak{M}$ and $\mathfrak{N}$, consisting respectively of m and n elements. The subsets $\mathfrak{M}_m$ and $\mathfrak{N}_n$ determine a certain matrix of m rows and n columns

$$\left\| a_{i_k \alpha_\lambda} \right\|_{\lambda=1,2,\dots,n}^{k=1,2,\dots,m} . \tag{1}$$

Each such matrix will be regarded as a point of the mn-dimensional arithmetic space R^{mn}. We shall consider the order in which the elements of the subsets $\mathfrak{M}_m$ and $\mathfrak{N}_n$ are taken to be immaterial, and therefore we shall assume that all $m!n!$ matrices obtained by permutations of the rows and columns of the matrix (1) describe one and the same physical situation. In what follows, transformations of the space R^{mn} corresponding to permutations of the rows and columns of the matrix (1) will be called canonical permutations of R^{mn}.

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We shall say that a physical structure of rank (m, n) is given on the sets $\mathfrak{M}$ and $\mathfrak{N}$ if the following conditions are satisfied:

A. There exists an analytic function of mn variables $\Phi(x_1, x_2, \dots, \dots, x_m)$ with a natural domain of definition, invariant with respect to canonical permutations of the space R^{mn}, such that the set $M = \{x_1, x_2, \dots, x_{mn}\} \subset R^{mn}$ defined by the equation

$$\Phi(x_1, x_2, \dots, x_{mn}) = 0, \tag{2}$$

is nonempty and the gradient of Φ is nonzero at least at one point of the set M.

B. The totality of all points $(a_{i_1\alpha_1}, a_{i_1\alpha_2}, \dots, a_{i_m\alpha_n})$ of the space R^{mn}, where $\{i_1, i_2, \dots, i_m\} = \mathfrak{M}_m \subset \mathfrak{M}$, $\{\alpha_1, \alpha_2, \dots, \alpha_n\} = \mathfrak{N}_n \subset \mathfrak{N}$, forms a subm–

set M, open relative to M. In particular, for any $\mathfrak{M}_m$ and $\mathfrak{N}_n$

$$\Phi(a_{i_1\alpha_1},\, a_{i_1\alpha_2}, \dots,\, a_{i_m\alpha_n}) = 0. \tag{3}$$

C. If equation (3) can be uniquely solved with respect to $a_{i_k\alpha_\lambda}$, then, considering $a_{i_k\alpha_\nu}$, where $\nu \neq \lambda$, and $a_{i_s\alpha_\lambda}$, where $s \neq k$, as two groups of parameters, we require that they enter essentially into f.

D. For every system of n numbers u_λ $(\lambda = 1, 2, \dots, n)$ satisfying the equation

$$\Phi(a_{11},\, a_{12}, \dots,\, a_{1n}; \dots;\, a_{m-1,1},\, a_{m-1,2}, \dots,\, a_{m-1,n};\, u_1, u_2, \dots, u_n) = 0,$$

where $a_{p\lambda} = a_{i_p\alpha_\lambda}$ $(p = 1, 2, \dots, m-1;\ \lambda = 1, 2, \dots, n)$, $i_p \in \mathfrak{M}_{m-1}$, $\alpha_\lambda \in \mathfrak{N}_n$, there is an element $i \in \mathfrak{M}$ such that $u_\lambda = a_{i\alpha_\lambda}$; analogously, for every system of m numbers v_μ $(\mu = 1, 2, \dots, m)$ satisfying the equation

$$\Phi(a_{11}, \dots, a_{1,n-1}, v_1;\ a_{21}, \dots, a_{2,n-1}, v_2; \dots;\ a_{m1}, \dots, a_{m,n-1}, v_m) = 0,$$

where $a_{p\lambda} = a_{i_p\alpha_\lambda}$ $(p = 1, 2, \dots, m;\ \lambda = 1, 2, \dots, n-1)$, $i_p \in \mathfrak{M}_m$, $\alpha_\lambda \in \mathfrak{N}_{n-1}$, there is an element $\alpha \in \mathfrak{N}$ such that $v_p = a_{i_p\alpha}$.

We shall also say that the functional dependence (3) defines a primary physical law of rank (m, n), invariant with respect to the choice of finite subsets $\mathfrak{M}_m$ and $\mathfrak{N}_n$ and realizable on the sets $\mathfrak{M}$ and $\mathfrak{N}$.

The requirement that relation (3) exist for any choice of m elements from the set $\mathfrak{M}$ and n elements from the set $\mathfrak{N}$ we call the principle of phenomenological symmetry. This principle most naturally expresses the fact of the equal status of all elements of the sets $\mathfrak{M}$ and $\mathfrak{N}$ with respect to the primary physical law of rank (m, n).

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In subsequent papers we shall show that the function $A : (i,\alpha) \mapsto a_{i\alpha}$ and the function $\Phi(u_1, u_2, \dots, u_{mn})$, which are a solution of the infinite system of functional equations (3) for given m and n, are determined uniquely, up to a certain transformation depending on the choice of a particular measuring operation and immaterial for the construction of the general theory. In other words, the requirement of the single fact of the existence of a primary physical law of rank (m,n), characterized by equality (3), makes it possible to determine both the admissible set of experimental data $a_{i\alpha}$ and the concrete expression for the physical law itself.

In this article we shall confine ourselves to the consideration of physical structures of the smallest possible rank $(2,2)$. In this simplest example we shall see what methods can be applied in solving the problem posed. In addition, the result obtained here gives us the possibility of analyzing the logical meaning hidden behind the traditional formulation of Newton' s second law, which will be done in the following article.

Thus, let the sets $\mathfrak{M}$ and $\mathfrak{N}$ and the function Φ satisfy conditions A, B, C, and D with $m = n = 2$. Instead of $i_1, i_2;\ \alpha_1, \alpha_2$ we shall henceforth write, respectively, $i, k;\ \alpha, \beta$. First of all, let us find sets $\mathfrak{M}_2 = \{0, 1\}$ and $\mathfrak{N}_2 = \{\bar{0}, \bar{1}\}$ such that

$$\frac{\partial \Phi}{\partial a_{i\alpha}}(a_{1\bar{1}},\, a_{1\bar{0}},\, a_{0\bar{1}},\, a_{0\bar{0}}) \neq 0. \tag{4}$$

Such a point $A_0 = (a_{1\bar{1}}, a_{1\bar{0}}, a_{0\bar{1}}, a_{0\bar{0}})$ can be found, for example, as follows. First choose an arbitrary point $A_1 = (a_{i_1\alpha_1},\, a_{i_1\beta_1},\, a_{k_1\alpha_1},\, a_{k_1\beta_1})$ at which the gradient of Φ is nonzero. Put $v_{i\alpha} = \partial\Phi/\partial a_{i\alpha}$. Then under canonical permutations in R^4 the vector $v = (v_{i\alpha}, v_{i\beta}, v_{k\alpha}, v_{k\beta})$, as is evident, is likewise subjected to canonical permutations. If $v \neq 0$, then among its canonical permutations there is at least one in which the first

coordinate is nonzero. Hence it is clear that among the points obtained from $(a_{i\alpha}, a_{i\beta}, a_{k\alpha}, a_{k\beta})$ by canonical permutations, there is at least one for which the partial derivative $\partial\Phi/\partial a_{i\alpha}$ is nonzero. We shall take this point as A_0. In view of (4), there exist $\varepsilon > 0$ and $\delta > 0$ such that the equation $\Phi(u_1, u_2, u_3, u_4) = 0$ is uniquely solvable with respect to u_1 in the domain

$$|u_1 - a_{11}| < \varepsilon, \quad |u_2 - a_{10}| < \delta, \quad |u_3 - a_{01}| < \delta, \quad |u_4 - a_{00}| < \delta,$$

$$u_1 = f(u_2, u_3, u_4).$$

By virtue of B, for any such u_1, u_2, u_3, u_4 there exist $i, k \in \mathfrak{M}$ and $\alpha, \beta \in \mathfrak{N}$ for which $u_1 = a_{i\alpha}, u_2 = a_{i\beta}, u_3 = a_{k\alpha}, u_4 = a_{k\beta}$. Let x and ξ be such that

$$|x - a_{10}| < \delta, \quad |\xi - a_{01}| < \delta, \tag{5}$$

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and put

$$u = f(x, \xi, a_{00}) = \varphi(x, \xi).$$

Applying condition D, one can prove that there exist i and α such that $u = a_{i\alpha}, x = a_{i0}, \xi = a_{0\alpha}$. Choose arbitrarily x_1, x_2, ξ_1, ξ_2 for which the inequalities (5) are satisfied.

Condition D allows us to conclude that there exist i, k, α, β such that

$$\varphi(x_1, \xi_1) = a_{i\alpha}, \quad \varphi(x_1, \xi_2) = a_{i\beta}, \quad \varphi(x_2, \xi_1) = a_{k\alpha}, \quad \varphi(x_2, \xi_2) = a_{k\beta},$$

whence we obtain the basic functional equation

$$\Phi[\varphi(x, \xi), \varphi(x, \eta), \varphi(y, \xi), \varphi(y, \eta)] = 0, \tag{6}$$

where $x = x_1,\ y = x_2,\ \xi = \xi_1,\ \eta = \xi_2$.

Differentiating (6) with respect to x, y, ξ, η, we obtain a system of 4 homogeneous linear equations in the unknowns $\partial\Phi/\partial u_1, \partial\Phi/\partial u_2, \partial\Phi/\partial u_3, \partial\Phi/\partial u_4$. By condition (4), at least one of the quantities $\partial\Phi/\partial u_i$ is nonzero, and therefore the determinant of the system is equal to zero:

$$\frac{\partial\varphi}{\partial x}(x, \eta)\frac{\partial\varphi}{\partial x}(y, \xi)\frac{\partial\varphi}{\partial \xi}(x, \xi)\frac{\partial\varphi}{\partial \xi}(y, \eta) -$$

$$-\frac{\partial\varphi}{\partial x}(x, \xi)\frac{\partial\varphi}{\partial x}(y, \eta)\frac{\partial\varphi}{\partial \xi}(y, \xi)\frac{\partial\varphi}{\partial \xi}(x, \eta) = 0.$$

Fixing y and η, this equation can be rewritten in the form

$$A(x)B(\xi)\frac{\partial\varphi}{\partial \xi}(x, \xi) - C(x)D(\xi)\frac{\partial\varphi}{\partial x}(x, \xi) = 0. \tag{7}$$

From the conditions postulated above it follows that there exists a point (x_0, ξ_0) such that

$$A(x_0)C(x_0) \neq 0, \quad B(\xi_0)D(\xi_0) \neq 0.$$

In a neighborhood of such a point (x_0, ξ_0), equation (7) can be written in the form

$$P(x)\frac{\partial\varphi}{\partial x}(x, \xi) - Q(\xi)\frac{\partial\varphi}{\partial \xi}(x, \xi) = 0.$$

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It is therefore not difficult to conclude that φ is representable in the form

$$\varphi(x,\xi) = \chi[R(x)S(\xi)],$$

where $\chi(x)$, $R(x)$, and $S(x)$ are arbitrary monotone functions of one variable.

We have

$$a_{i\alpha} = \varphi(x_i, \xi_\alpha) = \chi[R(x_i)S(\xi_\alpha)],$$

whence

$$\chi^{-1}(a_{i\alpha}) = R(x_i)S(\xi_\alpha).$$

Writing out analogous relations for $a_{i\beta}, a_{k\alpha}, a_{k\beta}$, we obtain that these quantities are connected by the relation

$$\Phi(a_{i\alpha}, a_{i\beta}, a_{k\alpha}, a_{k\beta}) = \chi^{-1}(a_{i\alpha})\chi^{-1}(a_{k\beta}) - \chi^{-1}(a_{i\beta})\chi^{-1}(a_{k\alpha}) = 0. \qquad (8)$$

We see that in a neighborhood of some point of the manifold M relation (8) is satisfied. Hence, in view of the analyticity of M, it follows that (8) is satisfied everywhere on M.

Thus, in the present case ($m = n = 2$), the set M is determined up to an arbitrary function $\chi(x)$ and, by the change of variables $b_{i\alpha} = \chi^{-1}(a_{i\alpha})$, is reduced to the manifold

$$\Phi(b_{i\alpha}, b_{i\beta}, b_{k\alpha}, b_{k\beta}) = b_{i\alpha}b_{k\beta} - b_{i\beta}b_{k\alpha} = 0.$$

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Note: Figure translations are in progress. See original paper for figures.

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