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Abstract

Full Text

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RESOLVING POWER OF A THREE-DIMENSIONAL HOLOGRAM AS AN OPTICAL IMAGING SYSTEM

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A hologram is usually considered as an optical system whose resolution is determined by a finite aperture transmitting a certain spectrum of spatial frequencies (*1*). With this approach (as, indeed, in most other optical situations), the implicit assumption is used that the distribution of the wavefront recorded on the hologram is an ideally plane section of the wave field. In obtaining three-dimensional holograms on Lippmann emulsions and various photochromic materials, the intensity distribution is recorded in a certain limited volume. In this case the usual concept of aperture cannot be used. It will be shown below that, in "reading" a three-dimensional hologram, the resolution in the image of object points is determined both by the transverse dimensions and by the thickness of the photosensitive layer.

Fig. 1. Scheme of hologram recording. A_1 and A_2 are arbitrary points of the object. The origin of coordinates for the object (O) is made to coincide with the origin of coordinates for the hologram (O')

Let us consider the process of recording, on a volume photosensitive element, the wave field from an object located in the far field (Fig. 1).

We shall assume that the wave field in the region of the hologram is a set of plane waves with wave vectors

$$\mathbf{K}_{A_i} = \frac{\mathbf{r}_{A_i}}{|\mathbf{r}_{A_i}|}\frac{1}{\lambda},$$

where $\mathbf{r}_{A_i}$ is the vector drawn from the origin of coordinates to an arbitrary point A_i of the object. In this case the structure of the hologram recording (we regard the detection as strictly quadratic) can be represented in the form of sinusoidal three-dimensional distributions of the type

$$1 + \cos\left[2\pi(\mathbf{K}_{A_1} - \mathbf{K}_{A_2})\tau + \varphi_{A_1 A_2}\right].$$

The process of reconstructing the image of an object point A reduces to diffraction of the "reading" wave, converging, for example, at the point A_i, on the corresponding harmonic distribution of darkening. Accordingly, the intensity distribution $I(\mathbf{r})$ in the image of any object point can be calculated on the basis of the theory of diffraction of electromagnetic waves by three-dimensional periodic structures.

In the kinematic approximation of this theory

$$I(\mathbf{r}) = I_0 \Phi(\mathbf{r}), \tag{1}$$

where I_0 is a constant quantity, and $\Phi(\mathbf{r})$ is Laue' s interference function (2);

$$\Phi = \left[\frac{\sin \pi \Delta K_{A_x} X}{\pi \Delta K_{A_x} X}\right]^2 \left[\frac{\sin \pi \Delta K_{A_y} Y}{\pi \Delta K_{A_y} Y}\right]^2 \left[\frac{\sin \pi \Delta K_{A_z} Z}{\pi \Delta K_{A_z} Z}\right]^2; \tag{2}$$

here

$$\Delta K_{A_x} = \frac{\Delta r_{A_x}}{|r_A|}\frac{1}{\lambda}\cos^2\theta_x, \qquad \Delta K_{A_y} = \frac{\Delta r_{A_y}}{|r_A|}\frac{1}{\lambda}\cos^2\theta_y,$$

ΔK_{A_z} can be expressed in terms of ΔK_{A_y}, ΔK_{A_z}, for small ΔK_{A_x} and ΔK_{A_y},

$$\Delta K_{A_z} = \left|\frac{K_{A_x}}{K_{A_z}}\right| \Delta K_{A_x} + \left|\frac{K_{A_y}}{K_{A_z}}\right| \Delta K_{A_y}. \tag{3}$$

It is essential that in formula (3) any variable ΔK_{A_x}, ΔK_{A_y}, ΔK_{A_z} is determined by the other two, since the equality

Fig. 2. Scheme of recording and reconstruction of holograms used in the work

$$K_x^2 + K_y^2 + K_z^2 = (1/\lambda)^2.$$

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Figure 2: Fig. 2. Scheme of recording and reconstruction of holograms used in the work

is always satisfied. In the present case, the variables with the smaller components of the wave vectors are chosen as independent (we assume that $|K_{A_z}| \gg |K_{A_x}|$ and $|K_{A_z}| \gg |K_{A_y}|$, see Fig. 1).

The first two factors in formula (2) describe the diffraction distribution of the image intensity of point A in an ordinary optical system with aperture dimensions X, Y (including for a plane hologram). Denoting this distribution by Φ_0 and substituting (3) into (2), we obtain

$$\Phi = \Phi_0 \left[\frac{\sin \pi Z \left(\left| K_{A_x}/K_{A_z} \right| \Delta K_{A_x} + \left| K_{A_y}/K_{A_z} \right| \Delta K_{A_y} \right)}{\pi Z \left(\left| K_{A_x}/K_{A_z} \right| \Delta K_{A_x} + \left| K_{A_y}/K_{A_z} \right| \Delta K_{A_y} \right)} \right]^2 . \qquad (4)$$

Thus, the distribution $I(\mathbf{r})$ depends on the hologram dimensions in the directions X, Y, and Z. In particular, if the inequality $X < Z$ is satisfied, then the distribution $I(\mathbf{r})$ for a three-dimensional hologram will differ noticeably from the corresponding distribution for a plane hologram (any optical system with the same linear aperture dimensions). Thus, in the case under consideration, the role of the aperture of the optical system is played by the entire volume of the photosensitive element. It is easy to show that the use of the indicated considerations testifies to the possibility of increasing the resolving power when recording on a volume photosensitive element. For given dimensions of the hologram and its orientation relative to the object, the angular dimensions of the diffraction image of a point are easily determined if one uses the construction in reciprocal space (see Fig. 3a) (3).

We carried out corresponding experiments using an LG-36 laser. Colored silver-halide crystals of dimensions $X = 20$ mm, $Y = 20$ mm, $Z_1 = 12$ mm and $Z_2 = 6$ mm were used.

The scheme of recording and reconstruction of the holograms is shown in Fig. 2. Recording was performed by two plane waves. During reconstruction of holograms-

gram 3 was illuminated by wave 1. The diffraction image of the reconstructed wave 2 was collected by lens 4, in whose focal plane the entrance slit of the photoelectronic multiplier 5 was placed. The intensity distribution $I(\Delta K_x, 0)$ in this plane was recorded. A slit of width $200\,\mu$ (shown by the dashed line in Fig. 2) served to limit the dimensions of the working region of the hologram in the X direction. For such an experimental arrangement, at $\theta = 11.3°$, formula (1) takes the form:

Figure 3

Figure 3: Figure 3

$$I(\Delta K_x, 0) = I_0 \left[\frac{\sin(\pi Z \cdot 0.196\, \Delta K_x)}{\pi Z \cdot 0.196\, \Delta K_x} \right]^2 \left[\frac{\sin(\pi X \cdot 0.976\, \Delta K_x)}{\pi X \cdot 0.976\, \Delta K_x} \right]^2 . \qquad (5)$$

Figure 3b presents the results of measurements of the intensity distribution in the diffraction image, obtained for two crystals of different thicknesses (curves 1 and 2, respectively, for Z_1 and Z_2). For comparison, the corresponding distribution for diffraction by a slit of the indicated dimensions is shown (curve 3). It is seen that, in agreement with the formulas given above, when volume holograms are used the width of the diffraction image is significantly narrowed. The positions of the first minima, calculated from formula (5), agree with the experimental data with sufficient accuracy.

Although all the reasoning has been carried out within the framework of kinematic theory, the result obtained will not change substantially when a more general dynamical theory of diffraction is used. Moreover, the conclusion that the resolving power can be increased when the wave field is recorded in a volume is quite general and appears to remain valid even if the plane-wave approximation is abandoned. To prove this assertion, let us recall the general theorem according to which a wave passing through a limited aperture can be described by a finite number of parameters N, which are called the degrees of freedom of the wave field and, being invariants of the wave field, determine the information capacity of the optical system ([4]). The number of degrees of freedom for the field inside a three-dimensional hologram is

$$N_{\mathrm{T}} = N_{\mathrm{P}}(1 + \Delta K_z Z \alpha), \qquad (6)$$

where N_{P} characterizes the wave field at the surface of a plane hologram whose dimensions coincide with the transverse cross section of the three-dimensional one, and $K_z^2 = K^2 - K_x^2 - K_y^2$, $\alpha < 1$. Since $N_{\mathrm{T}} > N_{\mathrm{P}}$, the resolution in reconstruc-

Fig. 3. Influence of hologram thickness on the intensity distribution in the reconstructed image of object points. **a**—Ewald construction; x^* and y^*—coordinate axes in reciprocal space; **b**—measured intensity distributions.

image of an object by means of a three-dimensional hologram may exceed the classical limit (for planar and linear apertures).

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