RELATION BETWEEN THE TEMPERATURE OF SURFACE WATER IN THE OCEAN AND THE POWER OF A TROPICAL HURRICANE

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Abstract

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RELATION BETWEEN THE TEMPERATURE OF SURFACE WATER IN THE OCEAN AND THE POWER OF A TROPICAL HURRICANE

In previous articles ([1,2]) a simplified scheme of the field of a tropical hurricane was considered, based on the idea of it as a heat engine "of the fifth kind," for which the heater is the waters of a warm surface current, and the refrigerator is all the surrounding half-space. Simple relations were derived, containing only algebraic functions, by which one can establish the connection between the various parameters of the hurricane field.

In the present article we shall try to find the relation between the temperature of the underlying ocean water and the power that is released during the condensation of vapor rising in the ascending currents within the "core" of the hurricane (from the "eye of the hurricane" to approximately 1/3 of the radius of the entire hurricane field). This approximate investigation will help to understand the cause of the unprecedented destruction caused in 1969 by Hurricane "Camille," which developed over the strongly overheated waters of the Caribbean Sea and the Gulf of Mexico.

We shall take as a basis formula (12), derived in work ([2]), and rewrite it in a form convenient for calculations; moreover, we shall express the power in energy units—in megawatts:

$$W = 2{,}63 f \lambda h a r_0 \left(\frac{b}{a} \frac{r_1}{r_0} - 1 \right) \cdot 10^{-5} \text{ MW}. \tag{1}$$

The notation here is as before: f is the moisture content of the near-surface layer of the atmosphere in g/m^3; λ is the heat of condensation of vapor in cal/g; h is the height of the near-surface layer, taken equal to 500 m; a, b are parameters in the equation describing the change in the radial component of the wind velocity as the distance r from the center of the field changes; r_0 is the radius of the "eye" of the hurricane; r_1 is the radius of the "core" of the hurricane, within which the vertical component w of the wind velocity is directed upward, and outside which it is directed downward.

As before, let us denote by ϑ_0 the air temperature above the outer boundary of the "eye," averaged over the entire column from the ocean surface to the upper boundary of the hurricane field, lying at an equivalent height of 7200 m, where the atmospheric pressure is equal to 100 mb. Let ϑ_1 similarly denote the averaged temperature of the air column above the outer boundary of the "core."

We shall express the difference between these temperatures in two ways. On the one hand, on the basis of (9) from work ([2]), we find:

$$\vartheta_0 - \vartheta_1 = N\left[\ln\frac{r_1}{r_0} + \ln\left(1 - \frac{b}{a}\ln\frac{r_1}{r_0}\right)\right]. \tag{2}$$

Here, for brevity, it is denoted

$$N = \frac{f\lambda}{\delta c_p \eta}\frac{h}{h_e}, \tag{3}$$

where δ is the density of air; c_p is its heat capacity at constant pressure; η is a numerical coefficient that appeared in ([2]) and, on the basis of literary sources, was taken equal to 0.4; h_e is the equivalent thickness of the atmosphere reduced to surface density, $h = 8000$ m.

On the other hand, on the basis of (3) from ([2]) one can write

$$\vartheta_0 - \vartheta_1 = 276\left[\frac{\log p_1 - \log p_0}{(\log p_0 - 2)(\log p_1 - 2)}\right]. \tag{4}$$

The wind speed in the hurricane field does not depend on the absolute values of the air pressure p_1 at the outer boundary of the "core" and the pressure p_0 at the outer boundary of the "eye," but only on the difference $p_1 - p_0$. Therefore, for convenience of calculation, let us put $p_1 = 1000$ mb in (4). Then this equation takes the simple form

$$\vartheta_0 - \vartheta_1 = 276\left(\frac{3 - \log p_0}{\log p_0 - 2}\right). \tag{5}$$

In paper ([2]), on the basis of (1), the supply of energy to a hurricane through the condensation of water vapor within the "core" was considered; this core occupied, in plan, the area of a circle with a radius of 225 km, i.e., with a diameter of about 4° of meridian arc. This is precisely the width of the belt occupied in the Atlantic Ocean by the North Equatorial Current, above which the great majority of Atlantic hurricanes begin their path (from the Cape Verde Islands, off the coast of Africa). There is reason to believe that the stability of a hurricane on its path westward, toward the Antilles, is due to the fact that the "core" does not extend beyond this strongly overheated belt of surface ocean waters. Therefore we shall retain for r_1, and also for r_0, their former numerical values used in ([1],[2]). There is no reason to change the ratio of the constants b/a in the equations describing the changes in the radial component u of the wind speed with changes in r: when the intensity of the hurricane changes, only the constants a and b themselves will change.

Under these conditions, (2) is written simply as

$$\vartheta_0 - \vartheta_1 = 1.39\,N, \tag{6}$$

where N is expressed by formula (3), which shows that N depends on f—on the vapor content in a cubic meter of air. According to (1), this same quantity also enters the expression for the power W. But, in addition, the parameter a enters there, characterizing the radial component of the wind speed at the outer boundary of the hurricane "eye." On the basis of our theoretical relation (10) from ([1]), confirmed by numerous measurements in nature, one can write an analogous relation between the quantity a (at a height of 500 m above the ocean) and the pressure difference $p_1 - p_0$:

$$a \approx 2.8\sqrt{p_1 - p_0}. \tag{7}$$

For $p_1 = 1000$ mb, the subtracted p_0 in this formula enters into (5) as an implicit function of the difference $\vartheta_0 - \vartheta_1$. Therefore the following procedure was adopted for the calculations: 1) by assigning different values of p_0, the quantity $\vartheta_0 - \vartheta_1$ was determined from (5); 2) from this quantity, on the basis of (6), N was determined; 3) knowing p_1 and p_0, the parameter a was determined from (7); 4) f was regarded as a certain time argument; it was substituted into (1), into which the computed parameter a, corresponding to this value of f, was also substituted. As in papers ([1],[2]), it was assumed that $r = 15$ km; $h = 500$ m; $\lambda = 539$ cal/g. As a result, a small table appeared, among whose rows there was a row of values of the power W, taken from the ocean within the limits of the hurricane "core" and supplied to feed the hurricane system by the release of the heat of condensation in the corresponding layers of air, as well as a row containing values of the time argument f. Dividing the values of f by the value of the relative humidity $e = 0.9$, which we take as constant, we obtained a row containing the amount q of vapor capable of saturating the reduced air (in g/m^3), at the as yet unknown air temperature, taken to be equal to the temperature of the surface water (this is natural in August and September in this region).

The values of this temperature were borrowed from the tables in Appendix I to the book ([3]). From the figures contained there, a curve was constructed relating q to the temperature ϑ_W. It remained only to interpolate along this curve and find the values of the power W corresponding to

to one or another value of the main argument ϑ_W. The following calculation results were obtained:

Figure 1

Figure 1: Figure 1

Table 1

ϑ_W, deg.	25	26	27	28	29	30	31	32
$W \cdot 10^{-8}$, MW	1.57	1.70	1.85	2.00	2.17	2.37	2.58	2.82

Now let us try to calculate the power $\mathcal{F}$ expended by the hurricane in overcoming the external forces of friction against the ocean surface. Let V_r denote the magnitude of the total wind-velocity vector at the very surface of the water. For a given angle of 18° between it and the tangential component, it exceeds this component at the water surface by 5%.

At the water surface the ratio of the parameters is $B_0/A_0 = B/A$, where the notation is the same as in $(1,2)$. Consequently, the power $\mathcal{F}$ can be expressed as follows:

Fig. 1

$$\mathcal{F} = 2\pi k\delta \int_{r_0}^{R} V_r^3 r\,dr = 2\pi k\delta \cdot 1.05^3 A_0^3 \int_{r_0}^{R} \left(1 - \frac{B}{A}\ln\frac{r}{r_0}\right)^3 r\,dr. \qquad (8)$$

After integration and collection of like terms this gives:

$$\mathcal{F} = 7.25 k\delta A_0^3 R^2 \Phi, \qquad (9)$$

where Φ denotes the sign-changing polynomial

$$\Phi = 0.764(1 - 1/x^2) - 0.53\ln x + 1.25(\ln x)^2 - 0.01(\ln x)^3. \qquad (10)$$

In turn $x = R/r_0$.

It was said above that it was possible to relate to the surface-water temperature ϑ_W the parameter a, which enters the expression for the radial velocity u at a height of 500 m above the ocean. Taking into account that at the water level itself the wind speed decreases by approximately 10%, A_0 can be related to a by the dependence

$$A_0 = 0.9A = 0.9\frac{75}{24.4}a,$$

whence it follows that

$$A_0^3 = 21a^3. \tag{11}$$

On the basis of studies of recent years it may be assumed that the coefficient of friction against the surface of a roughened ocean is less than against the land surface. We take the coefficient of surface friction to be $k = 0.0011$ dyn/cm^2 at a wind speed of 1 cm/s. In that case, with the existing relation among the various investigated quantities, one can calculate from (9) and (10) the dependence of the desired power $\mathcal{F}$ on the surface-water temperature ϑ_W. This dependence is represented by curve 1 in Fig. 1.

Such power would be absorbed by the external forces of air friction against the ocean surface if, in reality, at different distances from the center of the hurricane there arose those wind speeds which correspond to the power W taken from the surface water at its temperature ϑ_W. In fact, such power corresponds to the stationary state of the system only at a definite value of ϑ_W. There is reason to believe that the parameters specified in (1) correspond to $\vartheta_W = 28°$. We shall adopt this as a working hypothesis and denote by ε the efficiency coefficient characterizing the transition of part of the power W into the mechanical energy released at the ocean surface by the heat engine under study. Then, for the stationary state $\vartheta_W = 28°$, one may write $\mathcal{F} = \varepsilon W$.

The numerical value of ε in this particular case is determined on the basis of Table 1 and curve 1 in Fig. 1.

It turns out that $\varepsilon = 0.036$. But in general ε must depend on the temperature conditions. The heat engine of a tropical hurricane operates according to a very complex cycle, and an exact determination of this dependence is impossible. For our scheme it will not be too gross an error to assume that ε is proportional to the difference of the mean temperatures $\vartheta_0 - \vartheta_1$ of the air columns above the boundary of the "eye" and the boundary of the "core" (changes in the mean temperatures at more distant distances from the outer boundary of the field are very small). Having adopted this approximate dependence and knowing the values of $\vartheta_0 - \vartheta_1$ for various ϑ_W, we recalculated the values of ε, starting from the particular value given above, which corresponded to $\vartheta_W = 28°$. Multiplying the quantities W, taken from Table 1, by the calculated values of ε, it was not difficult to construct curve 2 in Fig. 1, expressing the relation between ϑ_W and the power εW that can be converted into the mechanical energy of friction at the ocean surface. As we see, at surface-water temperatures below 28° this power is insufficient: curve 2 here invariably lies below curve 1. The damping of those velocities that have been considered up to now in our approximate hurricane model is inevitable. On the contrary, at temperatures $\vartheta_W > 28°$ the ocean must supply the hurricane system with power exceeding that dissipated by friction. Therefore the wind speeds in the system must increase up to the stationary conditions: $\varepsilon W = \mathcal{F}$.

In 1969, in the Caribbean Sea and the Gulf of Mexico, a hurricane of unprecedented power developed. At one stage it passed* through the gulf over water with a temperature of about 32°, whereas, according to the map placed in (4), the mean multiyear surface-water temperature at this time of year in this area is less than 29°. As curve 2 in Fig. 1 shows, Hurricane "Camille" must have released mechanical energy more than one and a half times the mean multiyear norm characteristic of very powerful hurricanes in the Gulf of Mexico. The energy released by this hurricane was twice that characteristic of the most frequently occurring Atlantic hurricanes passing over a warm current with a surface temperature of 27°.

Our approximate scheme is not capable of quantitatively explaining the impossibility of the development of tropical hurricanes at $\vartheta_W < 27°$. However, a qualitative explanation is possible. It may proceed from consideration of the alternating-sign polynomial Φ (10), entering into expression (9) for $\mathcal{F}$ as a very important cofactor. This alternating-sign polynomial is very sensitive to changes in the quantity $x = R/r_0$. Up to now we have regarded both this quantity and all other parameter ratios as constant; meanwhile, at wind speeds possible above a water surface with $\vartheta_W < 27°$, the entire structure of the field must change; the values R/r_0 and their function Φ also change, which may lead to the elimination of the very conditions for the development of any vortex that has arisen up to hurricane strength. A lowering of the height of the upper boundary of the field encompassed by the vortex may also contribute to the elimination of these conditions.

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* According to data of the Hydrometeorological Center of the USSR.

Note: Figure translations are in progress. See original paper for figures.