STABILITY OF A FINITE-PRESSURE PLASMA WITH TRAPPED PARTICLES

PHYSICS

1970

SovietRxiv

Abstract

Full Text

UDC 533.9

PHYSICS

A. B. MIKHAILOVSKII, A. M. FRIDMAN

STABILITY OF A FINITE-PRESSURE PLASMA WITH TRAPPED PARTICLES

(Presented by Academician M. A. Leontovich, 5 III 1970)

In a plasma confined by a magnetic field that is nonuniform along the lines of force, there are particles which, in their motion, are reflected from regions with a large field strength—trapped particles—and particles moving without reflection —passing particles. B. B. Kadomtsev ([1]) showed that trapped particles can give rise to a plasma instability analogous to the flute instability. The result of ([1]) applies to a plasma of zero pressure, $\beta \equiv 8\pi p/B^2 \to 0$ (p is the plasma pressure, B is the magnetic-field strength). Here we investigate the possibility of the development of an instability of trapped particles in a finite-pressure plasma.

This problem can be considered in a mathematically rigorous way, as in ([2], [3]). We shall, however, confine ourselves to a qualitative analysis, following the spirit of Kadomtsev' s work ([1]).

As in ([1]), we assume the perturbations to be potential (this is justified in a rigorous treatment). The dispersion equation can be obtained by setting equal to zero the sum of the contributions from passing and trapped particles to the scalar dielectric permittivity ε_0. The passing particles are distributed according to Boltzmann. Therefore their contribution to ε_0, as in the case ([1]), is

$$\varepsilon_{0\,\mathrm{pass}} \simeq \frac{2}{k^2 d^2}, \tag{1}$$

where k is the transverse wave number; $d = (T/4\pi e^2 n_0)^{1/2}$ is the Debye radius; n_0, T are the plasma density and temperature.

The trapped particles, whose relative number we take to be of order $\alpha \ll 1$, move in the perturbed field in the same way as in the case of flute perturbations. Therefore

$$\varepsilon_{0\,\mathrm{tr}} \simeq \left\langle \frac{\omega - \omega_e^*}{\omega - \bar{\omega}_{De}} + \frac{\omega - \omega_i^*}{\omega - \bar{\omega}_{Di}} \right\rangle . \tag{2}$$

Here the bar denotes averaging over the particle trajectory with weight $1/v_\parallel$ ($v_\parallel$ is the longitudinal velocity), while the brackets $\langle\cdots\rangle$ denote averaging over the Maxwellian distribution. The quantities $\omega^*_\alpha \equiv T\mathbf{k}_\perp[\mathbf{e}_0, \nabla \ln n_0]/m_\alpha\omega_{B\alpha}$ are the gradient frequencies of the corresponding plasma components (electrons and ions), $\mathbf{e}_0 \equiv \mathbf{B}/B$, $\omega_{B\alpha} = eB/m_\alpha c$. The frequency of the magnetic drift of particles is $\omega_{D\alpha} \equiv \mathbf{k}\mathbf{V}_{D\alpha}$, where

$$\mathbf{V}_D = \frac{1}{\omega_B}\left[\mathbf{e}_0, \frac{v_\perp^2}{2}\nabla \ln B + v_\parallel^2(\mathbf{e}_0\nabla)\mathbf{e}_0\right]. \tag{3}$$

In the approximation $\beta \to 0$, this frequency is uniquely related to the radius of curvature of the lines of force R,

$$\omega_D = \mathbf{k}_\perp[\mathbf{e}_0, \mathbf{n}]\left(\frac{v_\perp^2}{2} + v_\parallel^2\right)/R\omega_B; \tag{4}$$

$\mathbf{n}$ is the principal normal to a line of force. In this case, from (1) and (2), in the approximation $\omega_D < \omega < \omega^*$, Kadomtsev' s result is obtained, indicating that instability with growth rate

$$\gamma \simeq (\alpha a/R)^{1/2}\omega^*, \tag{5}$$

where $a = |\nabla \ln n_0|^{-1}$.

Using the equilibrium condition $\nabla(p + B^2/8\pi) = -(\mathbf{B}\nabla)\mathbf{B}/8\pi$, we find that the approximation $\beta \to 0$ breaks down if

$$\beta > a/R. \tag{6}$$

For such β, the frequency of the magnetic drift ceases to depend on the curvature and can be written as

$$\omega_{D\alpha} = -\frac{\beta}{2}\frac{mv_\perp^2}{2T}\omega^*_\alpha. \tag{7}$$

Equating the sum of (1) and (2) to zero and putting $a/R < \beta < 1$, $\omega_D < \omega < \omega^*$, we arrive at a dispersion equation which, in contrast to (5), has solutions with real ω,

$$\omega \simeq (\alpha\beta)^{1/2}\omega^*. \tag{8}$$

This branch of stable perturbations exists for $\beta \lesssim \alpha$. At larger β, weakly damped branches of oscillations are absent altogether.

Thus, we have shown that the trapped-particle instability does not develop for $\beta > a/R$, condition (6). The reason for this is the stabilizing influence of the diamagnetic drift associated with finite β.

It is of interest to note the qualitatively different character of the influence of finite β on the trapped-particle instability and on the ordinary flute instability. In the case of the flute instability, along with the diamagnetic drift, the nonpotentiality of the perturbations is also important. These two effects completely compensate each other, so that the dispersion equation for flute perturbations remains one and the same both at large and at small β. In the case of the trapped-particle instability, the destabilizing effect of nonpotentiality is weakened by a factor α in comparison with the stabilizing effect of the diamagnetic drift. Therefore the plasma proves to be stable as soon as the diamagnetic drift outweighs the drift due to curvature.

Received
17 February 1970

REFERENCES

1. B. B. Kadomtsev, *Letters to JETP*, **4**, 15 (1966).
2. B. B. Kadomtsev, O. P. Pogutse, *JETP*, **51**, 1734 (1966).
3. M. N. Rosenbluth. *Phys. Fluids*, **11**, 869 (1968).

Note: Figure translations are in progress. See original paper for figures.