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PASSAGE THROUGH THE SELF-OSCILLATION ZONE OF A RIGID ROTOR ROTATING IN JOURNAL BEARINGS

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Self-oscillations of a rotor arise as a result of loss of stability of the equilibrium position of the journal in a journal bearing. Until now, the principal direction in improving the dynamics of such shafts has been measures aimed at shifting the boundary of loss of stability toward higher angular velocity. For this purpose ([1]), in the theory one determined the stability boundary of a loaded rotor on the basis of a linearized system of differential equations of shaft motion in variations, and changed the parameters of the rotor and bearing in order to widen the stability zone. In practice, vibration-resistant journal bearings are used for this purpose, improving the hydrodynamics of oil friction by abandoning the cylindrical cross section of the bearing.

Apart from the technological difficulties in manufacturing such bearings, the indicated approach permits only a slight shift of the boundary at which self-oscillations arise; as a rule, it is impossible to pass beyond it with a further increase of speed because of the high level and breadth of the self-oscillation zone. In addition, this method cannot eliminate the occurrence of forced oscillations of large amplitude caused by shaft unbalance.

Therefore another direction is now being developed ([2]). The self-oscillations of a rotor after loss of stability of the equilibrium state are investigated, which can be done only by considering the nonlinear system of differential equations of motion, and ways are determined to narrow the zone and lower the level of self-oscillations, making it possible to ensure an easy passage through the dangerous zone. In contrast to ([2]), where special bearings of complex shape are used for this purpose, in the present work elastic supports are used for this purpose, making it possible simultaneously to reduce the level of forced oscillations and the reactions between the rotor and the bearing ([3,4]).

1. The equations of motion of a symmetric rigid rotor in elastic journal bearings have the form (Fig. 1):

$$m_1\ddot{x}_1 + \beta_1\dot{x}_1 + cx_1 - P_e\cos\varphi - P_\varphi\sin\varphi = m_1 g,$$

$$m_1\ddot{y}_1 + \beta_1\dot{y}_1 + cy_1 - P_e\sin\varphi + P_\varphi\cos\varphi = 0,$$

$$m\ddot{x}_2 + \beta\dot{x}_2 + P_e\cos\varphi + P_\varphi\sin\varphi = \varepsilon\omega^2\cos\omega t + mg,$$

$$m\ddot{y}_2 + \beta\dot{y}_2 + P_e\sin\varphi - P_\varphi\cos\varphi = \varepsilon\omega^2\sin\omega t, \tag{1}$$

$$x_2 - x_1 = e\cos\varphi, \qquad y_2 - y_1 = e\sin\varphi,$$

$$\dot{e} = [(x_2 - x_1)(\dot{x}_2 - \dot{x}_1) + (y_2 - y_1)(\dot{y}_2 - \dot{y}_1)]/e,$$

$$\dot{\varphi} = [(\dot{y}_2 - \dot{y}_1)(x_2 - x_1) - (\dot{x}_2 - \dot{x}_1)(y_2 - y_1)]/e^2,$$

where m is the mass of the rotor; m_1 is the mass of both supports; e is the distance between the centers of the rotor and the support; c is the equivalent stiffness of the supports; ε is the rotor unbalance; x_1, y_1 are the coordinates of the center of the elastic support; x_2, y_2 are those of the rotor center; x_3, y_3 are those of the rotor center of mass; φ is the polar angle; β is the coefficient of external damping of the rotor; β_1 is the same for the supports.

Components of the reaction of the lubricating layer ([5])

$$\begin{aligned} P_e &= 12\mu L\frac{r^3}{\delta^2}[(\omega - 2\dot{\varphi})F_1(\chi) + 2\dot{\chi}F_2(\chi)], \\ P_\varphi &= 12\mu L\frac{r^3}{\delta^2}[(\omega - 2\dot{\varphi})F_3(\chi) + 2\dot{\chi}F_4(\chi)], \end{aligned} \tag{2}$$

where $\delta = R - r$ is the radial clearance; $\chi = e/\delta$ is the relative bearing clearance; r, L are the radius and length of the bearing; μ is the coefficient of dynamic viscosity.

$$\begin{aligned} F_1(\chi) &= \frac{2\chi^2}{(2+\chi^2)(1-\chi^2)}, & F_2(\chi) &= \frac{\pi\chi}{(2+\chi^2)(1-\chi^2)^{1/2}}, \\ F_2(\chi) &= \frac{1}{(1-\chi^2)^{3/2}}\left[\frac{\pi}{2} - \frac{8}{\pi(2+\chi^2)}\right], & F_4(\chi) &= \frac{2\chi}{(2+\chi^2)(1-\chi^2)}. \end{aligned} \tag{3}$$

Fig. 1. Computational diagram of a rotor rotating in elastic sliding bearings

Figure 1: Fig. 1. Computational diagram of a rotor rotating in elastic sliding bearings

2. We shall consider self-oscillations, taking the rotor to be vertical and balanced, the elastic supports to be massless and without damping ($\varepsilon = 0$;

Fig. 1. Computational diagram of a rotor rotating in elastic sliding bearings

$\beta_1 = 0$; $m_1 = 0$). Let us note that passage through the self-oscillation zone is especially relevant for a vertical rotor, since, in comparison with a loaded (horizontal) rotor, its self-oscillation zone and vibration level are always higher.

Introducing the complex numbers $\bar{z}_1 = x_1 + iy_1$; $\bar{z}_2 = x_2 + iy_2$; $\bar{e} = e(\cos\varphi + i\sin\varphi)$, we represent the first four equations of system (1) in the form

$$\ddot{\bar{z}}_2 + \gamma z_2 + \omega_0^2(\bar{z}_2 - \bar{e}) = 0, \qquad c(\bar{z}_2 - \bar{e}) = (P_e - iP_\varphi)e^{i\varphi}, \tag{4}$$

where $\gamma = \beta/m$, $\omega_0^2 = c/m$ is the square of the natural frequency of the rotor installed in elastic supports without taking into account the compliance of the oil layer.

Among the solutions of equations (4) we shall seek, following ([2],[6]), a certain class corresponding to established asynchronous precession, i.e., circular motion of the shaft axis with constant amplitudes and angular velocity Ω, not coinciding with the rotational speed ω. For a vertical rotor such an assumption, owing to circular symmetry, is natural and corresponds to experiment ([2]). Suppose further that at the initial instant of time the vector $\bar{z}_2$ coincided with the x-axis; then, taking into account the angle α between the vectors $\bar{z}_2$ and $\bar{e}$, the solution of system (4) should be sought in the form

$$\bar{z}_2 = Ae^{i\Omega t}, \qquad \bar{e} = \delta\chi e^{i(\Omega t + \alpha)}, \tag{5}$$

where A is the amplitude of oscillations of the rotor mass center.

The values of the radial and tangential components of the forces of the lubricating layer, taking into account the assumption made, take the form

$$P_e = 12\mu L\frac{r^3}{\delta^2}(\omega - 2\Omega)F_1(\chi); \qquad P_r = 12\mu L\frac{r^3}{\delta^2}(\omega - 2\Omega)F_3(\chi). \tag{6}$$

Substituting further (5) into equation (4), taking (6) into account, we obtain a system of nonlinear equations that makes it possible to find the relation between the amplitude of self-oscillations and the frequency:

Figure 2: Dependence of the amplitudes and width of the rotor self-oscillation zone on the angular velocity of rotation for $D = 0.1$ and $D = 0.2$, for $k = 5$ (1), 0.5 (2), and 0.05 (3).

Figure 2: Figure 2: Dependence of the amplitudes and width of the rotor self-oscillation zone on the angular velocity of rotation for $D = 0.1$ and $D = 0.2$, for $k = 5$ (1), 0.5 (2), and 0.05 (3).

$$[1-(\Omega/\omega_0)^2]\rho = \chi\cos\alpha,$$

$$D\frac{\Omega}{\omega_0}\rho = \chi\sin\alpha, \tag{7}$$

$$\rho\sin\alpha = 3k(\omega/\omega_0 - 2\Omega/\omega_0)F_1(\chi),$$

$$\rho\cos\alpha + \chi + 3k(\omega/\omega_0 - 2\Omega/\omega_0)F_3(\chi),$$

where $k = 4\mu L r^3/\omega_0 m\delta^3$ is a characteristic constant; $D = \gamma/\omega_0$ is a dimensionless coefficient; $\rho = A/\delta$ is the relative amplitude of oscillations of the rotor mass center.

Fig. 2. Dependence of the amplitudes and width of the rotor self-oscillation zone on the angular velocity of rotation for $D = 0.1$ and $D = 0.2$, for $k = 5$ (1), 0.5 (2), and 0.05 (3)

Eliminating α from the first two equations of system (7), we find the dependence of the rotor oscillation amplitude ρ on the eccentricity χ and the relative frequency of asynchronous precession Ω/ω_0:

$$\rho^2 = \frac{\chi^2}{[1-(\Omega/\omega_0)^2]^2 + D^2(\Omega/\omega_0)^2};$$

$$\frac{\chi}{6kF_1(\chi)}\left[\frac{1-\Omega^2/\omega_0^2}{[(1-(\Omega/\omega)^2]^2 + D^2(\Omega/\omega)^2} - 1\right] = \frac{\omega}{2\omega_0} - \frac{\Omega}{\omega_0}; \tag{8}$$

$$\frac{\chi}{6kF_3(\chi)}\frac{D\Omega/\omega_0}{[(1-\Omega^2/\omega_0^2)]^2 + D^2(\Omega/\omega)^2} = \frac{\omega}{2\omega_0} - \frac{\Omega}{\omega_0}.$$

The solution of system (8) was carried out graphically for 3 different values of the characteristic constants $k = 5;\ 0.5;\ 0.05$, for two values of the relative coefficient $D = 0.1;\ 0.2$. In Fig. 2, graphs are plotted of the dependence of the amplitudes of self-oscillations on the angular velocity of rotation. From these graphs it is seen that the characteristic constant k strongly affects the form of the curves $\rho = f(\omega/\omega_0)$. For small values of the characteristic constants k, the interval within which self-oscillations exist is so wide that it is practically impossible to determine the upper limit of their existence, and, consequently, to pass through it. With increasing k, the range of existence of self-oscillations narrows, and for sufficiently large

values of k the graph is transformed into a curve resembling a resonance curve. At large values of k, the interval in which self-oscillations exist is so small that it is possible not only to find the upper limit of their existence, but also to pass through it.

The amplitude of self-oscillations decreases sharply as the coefficient D increases. At the same time, the region of self-oscillations narrows. It follows from the curves that the maximum amplitude, for a fixed value of the coefficient D, remains constant for different values of k.

The self-oscillations of the rotor depend substantially on the characteristic constants of the system, k and D. Let us consider the values of these coefficients and analyze their variation with a change in the stiffness of the elastic rotor supports. Substituting the value of the natural frequency of rotor oscillations $\omega_0^2 = c/m$ into the expression for k and D, we obtain:

$$k = 4\mu L r^3/\sqrt{c}\sqrt{m}\delta^3; \qquad D = \gamma\sqrt{m}/\sqrt{c}. \tag{9}$$

It follows from expression (9) that, when the stiffness c of the elastic supports is decreased while the remaining parameters of the rotor and bearing remain unchanged, the characteristic constant k increases and, consequently, the interval in which self-oscillations exist becomes narrow; it is then possible not only to determine the upper limit of their existence, but also to pass through it, just as the critical speeds of the rotor are passed through. It follows further from expressions (9) that, when the stiffness c of the supports is decreased while the remaining parameters of the system remain unchanged, the relative coefficient D increases and, consequently, the amplitude of self-oscillations decreases sharply. Thus, mounting the rotor in elastic supports, with a proper choice of their stiffness, makes it possible to ensure passage of the rotor through the self-oscillation zone, beyond which operation of the rotor with small vibration amplitudes is possible.

The upper limit of the stiffness of the elastic supports, at which operation of the rotor beyond the boundary of self-oscillations with small vibration amplitudes is possible, is determined by the formula

$$c_* \leqslant 16\mu^2 L^2 r^6/k^2 m\delta^6. \tag{10}$$

Let us note that replacing the compliance of the supports by compliance of the shaft is, as a rule, impossible because of strength requirements imposed on the shaft, the need to ensure normal thermal operating conditions of the bearing, and the danger of edge pressure appearing near the end regions of the liner.

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Note: Figure translations are in progress. See original paper for figures.