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Abstract

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PHYSICS

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STATISTICAL FEATURES OF MULTIQUANTUM PHOTOREGISTRATION OF RADIATION

(Presented by Academician A. M. Prokhorov on 16 IV 1970)

It is known that the correlation properties of optical fields are studied mainly from data on the probability distribution of photoelectric counts (see, for example, ([1,2])). The theoretical and experimental studies carried out so far on the relation between the statistics of a radiation field and the statistics of a photocurrent have, however, concerned only the case of the one-quantum photoeffect, when the probability of appearance of a photoelectron is proportional to the square of the value of the radiation-field amplitude (the case of so-called "quadratic" detection). Recent theoretical and experimental studies (see the review ([3])) have shown that an elementary act of the external multiquantum photoeffect can, in principle, be a detector of the order and degree of coherence of radiation, since the amplitude values of the photocurrent depend on the correlation properties of the electromagnetic field that produces it.*

Much more important information on the coherent properties of the radiation field is contained, nevertheless, not in the amplitude values of the multiquantum photocurrent, but in its fluctuation characteristics, which until now have not been studied either theoretically or experimentally. The present work is devoted to an evaluation of some basic features of the statistics of a multiquantum photocurrent caused by the "nonquadraticity" of the photodetector.

It can be shown ([4]) that, within the framework of the semiclassical theory of photoelectric registration of radiation, the probability $p_k(n,t,T)$ of counting n photoelectrons appearing above a unit surface area of the target in the process of a multiquantum photoeffect of k-th order in the time interval from t to $t+T$, for stationary and ergodic fields, is given by the expression**

$$p_k(n,t,T) \equiv p_k(n,T) = \int_{(W)} \frac{(\alpha_k W_k)^n}{n!} \exp[-\alpha_k W_k] P(W)\, dW, \qquad (1)$$

where

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$$W_k \equiv W_k(T) = \int_0^T [I(t')]^k \, dt', \qquad W = \int_0^T I(t') \, dt', \tag{2}$$

I is the radiation intensity, and α_k is the quantum yield of the k-th order photoeffect. In equation (1), which is a generalization of Mandel' s well-known formula ([6]) to the case of multiquantum photoemission, $P(W)\,dW$ denotes

* It should be noted that the rapid progress in techniques for registering weak photocurrents and the improvement of photocathodes have led to the fact that, for example, the two-quantum photoeffect is observed at an incident radiation intensity $I \gtrsim 10^3$ W $\cdot$ cm^{-2}, which corresponds to irradiating the target with focused radiation from a source having radiation power in the spectral line $P \simeq$ mW.

** In work ([5]) it was shown that the results of the semiclassical approach, in which the field is described classically and the elementary act of the photoeffect quantum-mechanically, coincide with the results of a purely quantum-mechanical treatment as long as the influence of the measuring devices on the radiation field can be neglected.

the probability that during the time interval ΔT the classical integral intensity of the radiation W varies within the limits from W to $W + dW$. The operation of ensemble averaging $W(T)$ in expression (1) thus leads to a deviation of the photoelectron distribution from the Poisson distribution; moreover, precisely this deviation is a measure of the fluctuation (correlation) properties of the radiation field.

To determine the factorial moments of the m-th order $M_k^{(m)}$, corresponding to the distribution of the k-quantum photocurrent $p_k(n,T)$ and directly characterizing the fluctuation properties of the radiation field (7), let us construct the generating function $G_k(r)$ for the distribution $p_k(n,T)$. Let (8)

$$G_k(r) = \sum_{n=0}^{\infty} p_k(n,T) r^n = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} (1-r)^m M_k^{(m)}, \tag{3}$$

where

$$M_k^{(m)} = \sum_{n=0}^{\infty} p_k(n,T)[n(n-1)\dots(n-m+1)]$$

is the factorial moment of order m. After simple transformations for the distribution $p_k(n,T)$ specified by expression (1), we have

$$G_k(r) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} (1-r)^m \int_{(W)} [\alpha_k W_k]^m P(W) \, dW$$

and, consequently,

$$M_k^{(m)} = \langle [\alpha_k W_k]^m \rangle. \tag{4}$$

Thus, the initial moment $\langle (\alpha_k W_k)^m \rangle$ of the quantity $\alpha_k W_k$, averaged over the distribution $P(W)$, is equal to the m-th factorial moment $M_k^{(m)}$ of the distribution $p_k(n,T)$, and

$$\langle n^m \rangle = a_1^{(m)} \langle \alpha_k W_k \rangle + a_2^{(m)} \langle (\alpha_k W_k)^2 \rangle + \dots + a_m^{(m)} \langle (\alpha_k W_k)^m \rangle, \tag{5}$$

where

$$a_j^{(m)} = \sum_{i_1=1}^{m-1} \sum_{i_2=1}^{i_1-1} \dots \sum_{i_{j-1}=1}^{i_{j-2}-1} \binom{m-1}{i_1} \binom{i_1-1}{i_2} \dots \binom{i_{j-2}-1}{i_{j-1}},$$

$$j = 2, 3, \dots, m; \qquad a_1^{(m)} = 1,$$

and

$$\langle (\alpha_k W_k)^m \rangle = b_1^{(m)} \langle n \rangle + b_2^{(m)} \langle n^2 \rangle + \dots + b_m^{(m)} \langle n^m \rangle, \tag{6}$$

where

$$b_j^{(m)} = (-1)^{m-j} \sum (x_1 x_2 \dots x_{m-j}); \qquad j = 1, 2, \dots, m-1; \qquad b_m^{(m)} = 1$$

(the summation in the last expression is carried out over the entire set of successively possible combinations of the products $x_1 x_2 \dots x_{m-j}$, counted only once).

The possibility of studying higher correlation functions of radiation fields with the aid of multiquantum photodetectors becomes clear if expression (4), using relation (2), is written in the form

$$M_k^{(m)} = (\alpha_k)^m \int_0^T \dots \int_0^T \langle [I(t_1)]^k \cdots [I(t_m)]^k \rangle \, dt_1 \dots dt_m. \tag{7}$$

Since the integrand in relation (7) is, as is known ([9]), a correlation function $\Gamma^{(km)}$ of order km, it is clear that it can be found by various methods under the condition $km = \text{const}$. In particular, the use of a k-quantum photodetector makes it possible to increase the order of the correlation functions under study, in comparison with the case of one-quantum detection, by a factor of k, if one is

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concerned with measuring the amplitude values of the photocurrent corresponding to the first moment $M_k^{(1)} = \langle n_k \rangle$. The study of the fluctuation properties of a multiquantum photocurrent (for example, the variance) in turn opens up the possibility of experimentally studying correlation functions $\Gamma^{(s)}$ of order $2k$. Indeed (see expression (5)):

$$\left\langle (\Delta n_k)^2 \right\rangle = \left\langle (n_k - \langle n_k \rangle)^2 \right\rangle = \langle n_k \rangle + (\alpha_k)^2 \left\langle (\Delta W_k)^2 \right\rangle, \tag{8}$$

where the term $(\Delta W_k)^2 \sim \Gamma^{(2k)}$ takes into account the fluctuation of the classical wave fields.

Let us illustrate the possibility of using multiquantum photodetectors to study the statistical properties of the radiation field when $\Delta T < \tau_c$, where $\tau_c \sim 1/\Delta\nu$ is the coherence time of the incident radiation. In this case (see expression (2))

$$W_k(T) \simeq I^{kT}, \qquad W(T) \simeq IT, \tag{9}$$

and therefore

$$M_k^{(m)} = (\alpha_{kT})^m \int_{(I)} I^{km} P(I)\, dI = (\alpha_{kT})^m \langle I^{km} \rangle. \tag{10}$$

The expressions for the factorial moments $M_k^{(m)}$ and the fluctuations in the number of particles $\langle (\Delta n_k)^2 \rangle$ for particular cases of the distributions $P(I)$, given by the relations

$$P(I) = \frac{2}{\pi I_0} \frac{1}{1 + \operatorname{erf} w} \exp\left[-\left(w - \frac{I}{\sqrt{\pi I_0}} \right)^2 \right], \qquad I > 0, \tag{11a}$$

$$P(I) = \frac{1}{\gamma \langle I \rangle} \left\{ \exp\left[-\frac{2I}{(1+\gamma)\langle I \rangle} \right] - \exp\left[-\frac{2I}{(1-\gamma)\langle I \rangle} \right] \right\}, \tag{11b}$$

$$P(I) = \frac{N}{\langle I_n \rangle} \left(\frac{I}{\langle I_c \rangle} \right)^{(N-1)/2} \exp\left[-\frac{I + \langle I_c \rangle}{\langle I_n \rangle} N \right] J_{N-1} \left(\frac{2N\sqrt{I \langle I_c \rangle}}{\langle I_n \rangle} \right), \tag{11c}$$

have, respectively, the form*

$$M_k^{(m)} = (km)!\, \frac{\exp\left(-\frac{w^2}{2}\right)}{1 + \operatorname{erf} w} \left[\alpha_{kTI} 0^k \right]^m \left(\frac{\pi}{2} \right)^{(km-1)/2} D_{-km-1}(-\sqrt{2}w),$$

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$$\langle(\Delta n_k)^2\rangle = \langle n_k\rangle + \langle n_k\rangle^2 \left[\frac{(2k)!}{(k!)^2}\left(\frac{\pi}{2}\right)^{1/2}\exp\left(\frac{w^2}{2}\right)[1+\operatorname{erf} w]\,\frac{D_{-2k-1}(-\sqrt{2}w)}{[D_{-k-1}(-\sqrt{2}w)]^2} - 1\right], \tag{12a}$$

* Distribution (11a) corresponds to the model of a radiation field generated by a nonlinear Van der Pol oscillator ([10]), where w is a numerical parameter characterizing laser operation in the "below-threshold" ($w < 0$) and "above-threshold" ($w > 0$) regimes, and I_0 is the intensity of the laser radiation at the generation threshold, when $w = 0$. Distribution (11b) corresponds to a thermal source with degree of polarization of the radiation equal to γ ([1]). Distribution (11c) is a generalization, introduced in ([11]) for the case $N = 2$, of the distribution $P(I)$ describing the total radiation field, which is a superposition of N unphased modes and thermal radiation with mean intensity $\langle I_n\rangle$ (the mean intensity of the coherent radiation is denoted by $\langle I_c\rangle$).

$$M_k^{(m)} = \frac{(km)!}{2^{km}}[\alpha_k T\langle I\rangle^k]^m \frac{1}{2\gamma}\left[(1+\gamma)^{km+1} - (1-\gamma)^{km+1}\right],$$

$$\langle(\Delta n_k)^2\rangle = \langle n_k\rangle + \langle n_k\rangle^2\left[\frac{(2k)!}{(k!)^2}\,2\gamma\,\frac{(1+\gamma)^{2k+1}-(1-\gamma)^{2k+1}}{\left[(1+\gamma)^{k+1}-(1-\gamma)^{k+1}\right]^2} - 1\right], \tag{12 }$$

$$M_k^{(m)} = \frac{\Gamma(km+N)}{\Gamma(N)}\exp\left[-N\frac{\langle I_c\rangle}{\langle I_n\rangle}\right]\left(\frac{\alpha_k T\langle I_n\rangle^k}{N^k}\right)^m \Phi\left(km+N, N, N\frac{\langle I_c\rangle}{\langle I_n\rangle}\right),$$

$$\langle(\Delta n_k)^2\rangle = \langle n_k\rangle + \tag{12 }$$

$$+\langle n_k\rangle^2\left[\exp\left(N\frac{\langle I_c\rangle}{\langle I_n\rangle}\right)\frac{\Gamma(2k+N)\Gamma(N)}{[\Gamma(k+N)]^2}\,\frac{\Phi\left(2k+N, N, N\frac{\langle I_c\rangle}{\langle I_n\rangle}\right)}{\left[\Phi\left(k+N, N, N\frac{\langle I_c\rangle}{\langle I_n\rangle}\right)\right]^2} - 1\right],$$

where $D_s(z)$ is the parabolic-cylinder function, $\Phi(a, b, x)$ is the confluent hypergeometric function, $\operatorname{erf} x$ is the probability integral, $\Gamma(z)$ is the gamma function, and J_{N-1} is the modified Bessel function of order $N-1$.

Relations (12), for $k = 1$ and $N = 1$, reduce to the known expressions obtained for the case of single-quantum detection in ([2], [12]), and for $N \to \infty$ to the expressions

$$M_k^{(m)} = (km)![\alpha_k T\langle I\rangle^k]^m,$$

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$$\langle (\Delta n_k)^2 \rangle = \langle n_k \rangle + \langle n_k \rangle^2 \left[\frac{(2k)!}{(k!)^2} - 1 \right],$$

which are characteristic of the case of k-quantum detection of radiation with the exponential intensity distribution

$$P(I) = \frac{1}{\langle I \rangle} \exp \left(-\frac{I}{\langle I \rangle} \right),$$

inherent in a thermal source and constituting a special case of distribution (11) for $\gamma = 1$.

The relations obtained show that a direct study of the fluctuation characteristics of a multiquantum photocurrent makes it possible to investigate correlation functions of high orders. In turn, the use of more complicated photodetection schemes, which were not considered here (experiments of the Hanbury Brown –Twiss type, various schemes of photoelectric recording with time delays, etc.), will in principle make it possible to study experimentally the properties of correlation functions of still higher orders, which is especially important when the fields under study differ substantially from thermal fields and, consequently, correlation functions of higher orders are not reducible to correlation functions of lower orders, as is the case for thermal radiation.

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