INTEGRABILITY OF THE MAJORANT OF PARTIAL SUMS OF A TRIGONOMETRIC SERIES WITH QUASICONVEX COEFFICIENTS

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Abstract

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MATHEMATICS

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INTEGRABILITY OF THE MAJORANT OF PARTIAL SUMS OF A TRIGONOMETRIC SERIES WITH QUASICONVEX COEFFICIENTS

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Let the numbers a_k, $k = 0, 1, 2, \dots$, tend to zero and form a quasiconvex sequence, i.e. the series converges

$$\sum_{k=1}^{\infty} k\left|\Delta^2 a_{k-1}\right|, \qquad \text{where } \Delta^2 a_{k-1} = a_{k-1} - 2a_k + a_{k+1}.$$

Then the series

$$\frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos kx, \qquad \sum_{k=1}^{\infty} a_k \sin kx \tag{1}$$

converge for all $x \in (0, \pi]$ to functions continuous for these x, which we shall denote respectively by $f(x)$ and $g(x)$.

It is known that under the assumptions made $f \in L[0, \pi]$ (([1]; ([2], § 5.12), and for the integral of the modulus of the function g the estimate ([3]) is valid

$$\int_{\varepsilon}^{\pi} |g(x)|\, dx = \sum_{k=1}^{[1/\varepsilon]} \frac{|a_k|}{k} + O(1), \tag{2}$$

whence it follows that $g \in L[0, \pi]$ if and only if the series

$$\sum_{k=1}^{\infty} \frac{|a_k|}{k}$$

converges.

Consider the question of integrability of the majorants of the partial sums of the series (1), i.e. of the functions

$$f^*(x) = \max_n \left| \frac{a_0}{2} + \sum_{k=1}^{n} a_k \cos kx \right|, \qquad x \neq 0,$$

$$g^*(x) = \max_n \left| \sum_{k=1}^{n} a_k \sin kx \right|.$$

Theorem. Let $\{a_k\}$ be a quasiconvex sequence of numbers tending to zero. Then, as $\varepsilon \to +0$, the estimates

$$\int_\varepsilon^\pi f^*(x)\,dx = \sum_{k=1}^{[1/\varepsilon]} \frac{1}{k} \max_{n \geq k} |a_n| + O(1), \tag{3}$$

$$\int_\varepsilon^\pi g^*(x)\,dx = \sum_{k=1}^{[1/\varepsilon]} \frac{1}{k} \left(|a_k| + \max_{n \geq k} |a_n| \right) + O(1). \tag{4}$$

In particular, each of the functions f^* and g^* belongs to $L[0,\pi]$ if and only if the series

$$\sum_{k=1}^{\infty} \frac{1}{k} \max_{n \geq k} |a_n| \tag{5}$$

converges.

From estimates (2) and (4) we conclude that, for series with quasiconvex coefficients,

$$\int_\varepsilon^\pi g^*(x)\,dx \geq 2 \int_\varepsilon^\pi |g(x)|\,dx + O(1), \tag{6}$$

and if one additionally assumes that a_k decrease monotonically, then

$$\int_\varepsilon^\pi g^*(x)\,dx = 2 \int_\varepsilon^\pi |g(x)|\,dx + O(1). \tag{7}$$

Let us note that from the known results for series with monotone coefficients it follows that relation (7) holds even without the assumption of quasiconvexity of $\{a_k\}$, under the sole condition that a_k decrease monotonically.

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References

[1] A. Kolmogoroff, *Bull. Acad. Polon., Ser. A., Sci. Math.*, **83** (1923).
[2] A. Zygmund, *Trigonometric Series*, Moscow–Leningrad, 1939.
[3] S. A. Telyakovskii, *Matem. sbornik*, **63** (105), No. 3, 426 (1964).

Note: Figure translations are in progress. See original paper for figures.