NONLINEAR PROBLEM OF IMPINGEMENT OF ANTIPARALLEL FLOWS OF RAREFIED PLASMA

PHYSICS

1970

SovietRxiv

Abstract

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UDC 533.95

PHYSICS

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NONLINEAR PROBLEM OF IMPINGEMENT OF ANTIPARALLEL FLOWS OF RAREFIED PLASMA

(Presented by Academician R. Z. Sagdeev on 23 X 1969)

1. In the present work, using numerical methods within the framework of a one-dimensional model, we solve the nonstationary problem of nonlinear electrostatic oscillations in interpenetrating flows of rarefied plasma. As is known ([1]), in problems of this kind, under certain conditions one may expect solutions of the collisionless-shock-wave type.

Let a flow of rarefied electron–proton plasma move along the x-axis with mean velocity $-U_0$. In the plane $x = 0$, at the time $t = 0$, a collision occurs with an identical flow having velocity U_0. For $t \leqslant 0$, a Maxwellian velocity distribution is assumed for each component; at the same time the case of a strongly nonisothermal plasma with $T_e \gg T_i$ is considered. Restricting ourselves by the additional condition $U_0 \ll c_{Te} = (T_e/m_e)^{1/2}$, we assume that the electrons are Boltzmann-distributed and that $n_e = n_0 \exp(\varepsilon\varphi/T_e)$ ($\varphi(x,t)$ is the electrostatic potential, ε is the magnitude of the electron charge, $n_0 = n_{0i} = n_{0e}$ is the particle density in the unperturbed flow).

In the simplest case $T_i = 0$, in the region of two-flow motion the dispersion relation for longitudinal ion–ion electrostatic oscillations, obtained from linear theory, has the form

$$\omega^2/k^2 = U_s^2 \left([1 \pm \sqrt{1 + 4M^2(2 + k^2D_e^2)}]/(2 + k^2D_e^2) + M^2 \right), \tag{1}$$

according to which the condition for the development of instability is the requirement $M^2 < 2/(2 + k^2D_e^2)$ ($U_s = (T_e/m_i)^{1/2}$ is the ion-sound speed, $M = U_0/U_s$, $D_e = (T_e/4\pi n_0\varepsilon^2)^{1/2}$).

2. The behavior of the system under consideration is described by the Vlasov equations

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} - \frac{\varepsilon}{m_i}\frac{\partial \varphi}{\partial x}\frac{\partial f}{\partial u} = 0, \tag{2}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = 4\pi n_0 \varepsilon \left[a \exp(\varepsilon\varphi/T_e) - \int_{-\infty}^{\infty} f\, du \right] \tag{3}$$

with boundary conditions

$$f(0,u,t) = f(0,-u,t); \qquad \partial\varphi(0)/\partial x = 0, \qquad \varphi(\infty) = 0$$

and the initial condition

$$f(x,u,0) = f_0(x,u).$$

Here $f(x,u,t)$ is the ion distribution function, u is the x-component of the velocity. The factor a, taking the values a_1, a_2, corresponds to two possible formulations of the given problem. The first of them corresponds to the joint motion of ions and electrons ([2]), when $a = a_1$ is chosen in such a way that, in the absence of an initial perturbation, no charge separation occurs (for example, in the case $T_i = 0$, $a_1 = 1$ in the region of one-

flow motion and $\alpha_1 = 2$ in the mixing region). The second formulation of the problem corresponds to a uniform distribution of electrons throughout space ($a = \alpha_2 = 1$).

3. To solve the nonlinear system (2), (3), a discrete plasma model is used, in which the phase space (x,u) is divided into NQ cells by dividing the configuration space (in x) and momentum space (in u) respectively into N and Q intervals. The initial ion distribution

$$f_0(u) = (m_i/2\pi T_i)^{1/2} \exp[-m_i(u+U_0)^2/2T_i]$$

is approximated by the function

$$\bar{f}_0(u) = \sum_{q=1}^{Q} a_q \delta[u + (U_0 - u_q)], \tag{4}$$

where a_q, u_q are found from the conditions of equality of the moments

$$\int_{-\infty}^{\infty} u^{2r} f_0\, du = \int_{-\infty}^{\infty} u^{2r} \bar{f}_0\, du$$

$$(r = 0, 1, \dots, Q-1;\ a_q = a_{Q-q},\ u_q = -u_{Q-q},\ u_{(Q+1)/2} = 0,\ Q \text{ odd}).$$

Fig. 1

Fig. 3: phase-space distribution plot with axes u/U_0 and x/D_e; legend a, b.

Figure 1: Fig. 3: phase-space distribution plot with axes u/U_0 and x/D_e; legend a, b.

A self-consistent solution is obtained by joint numerical integration, on each time interval, of $Q(N+1)$ characteristic equations for (2), which simultaneously are equations of motion for the boundaries of the ion layers, and of the nonlinear Poisson equation (3), which is solved by the relaxation method. The main criterion for the accuracy of the calculation is the condition of constancy of the total energy of the system

$$E_{\text{pot}}(t) + E_{\text{kin}}(t) = E_0.$$

Fig. 2

The spatial grid is movable and at each fixed instant is determined by superposing the coordinates of the boundaries of all NQ ion layers. The density of positive charges in a spatial cell is found by summing over all Q energy groups and in each of them is proportional to the spatial density of the corresponding ion layers.

Various modifications of the so-called charged-layer method have previously been used for other problems in works ([3–5]). We note that in the proposed variant the combination of the approximation of the initial distribution with weight (4) with a dynamic Lagrangian grid makes the ion

layers bear the maximum informational load. At any moment it is possible to reconstruct the distribution function approximately.

4. In the figures presented, for two problems: $a = a_1$ (Figs. 1-3) and $a = a_2$ (Fig. 4), the results are given of a calculation of one of the variants considered, with characteristic dimensionless parameters $T_i/T_e = 5 \cdot 10^{-6}$, $M = 0.16$. In this case $U_0/c_{Ti} \approx 70$, $\lambda_{pi} = 2\pi U_0/\omega_{pi} = 0.95 D_e$ $(\omega_{pi} =$

Fig. 3

$= 2\pi/\tau_{pi} = \sqrt{4\pi n_0 e^2/m_i})$; the increment of ion-ion oscillations according to linear theory ([6]) is $\gamma \approx 0.4$. As the initial perturbation, "thermal noise" with energy of order $0.01\,T_i$ was introduced into the initial velocity distribution, specified in the form of a superposition of 10 sinusoids with different wavelengths. The model parameters are: $Q = 5$, the number of particles per Debye radius at the initial moment is 105.

In Fig. 1 the time dependence is given of the potential energy

$$E_{\text{pot}} = -\frac{1}{8\pi}\int_0^{x_{\text{fr}}} \left(\frac{\partial\varphi}{\partial x}\right)^2 dx,$$

Fig. 4

Figure 2: Fig. 4

referred to the total energy $E_0 = n_0(m_{iU}0^2/2 + T_i)x_{\text{fr}}$ (x_{fr} is the coordinate of the boundary of the mixing zone). The intensive development of nonlinear oscillations begins after $t = 2\tau_{pi}$, and by the time $t = 4\tau_{pi}$ saturation has practically set in. The energy of the electrostatic field is then about 10% of the total energy of the flow. As is seen from the next two figures, in which, at the moment $t = 5.25\,\tau_{pi}$, the spatial density of ions (Fig. 2) and the distribution in phase space of particles of the direct (a) and counter (b) flows (Fig. 3) are shown, the solution obtained reaches a quasistationary regime and has the form of a collisionless shock wave with a front thickness $\Delta \approx \lambda_{pi} \sim D_e$. It is clearly seen

(Fig. 2), as small-scale oscillations caused by the "bunching" of ions are modulated by the fundamental harmonic of the ion-ion oscillations. In this case, behind the shock-wave front (Fig. 3) there occurs strong scattering and thermalization of ions on electrostatic oscillations, accompanied by a severalfold change in the velocity of small groups of particles.

An entirely different picture is observed, for the same physical parameters, in the problem with a homogeneous distribution of electrons ($a = a_1 = 1$,

Fig. 4

Fig. 4). At the instant $t = 0$, the sharp increase in ion density upon collision leads to the formation, near $x = 0$, of a potential barrier

$$e\varphi \approx 3.5\,m_i U_0^2,$$

which generates an ion-acoustic wave propagating into the unperturbed flows with a phase velocity of order U_s. In this case the flows themselves, with the exception of a small leading group of fast particles, practically do not penetrate into one another.

Both problems considered were compared with supersonic calculations for $T_i/T_e = 5 \cdot 10^{-4}$ and, correspondingly, $M = 1.6$, with the ratio U_0/c_{Ti} unchanged. In the problem with $a = a_1$, in accordance with estimate (1), only a weak instability was observed, due to the relative motion of different thermal groups of ions, with the overall laminar character of the motion. In the problem with $a = a_2 = 1$, the supersonic flows freely penetrate into one another, generating in the mixing zone a wave with mean energy density

$$\frac{1}{8\pi}\langle(\partial\varphi/\partial x)^2\rangle \approx 10^{-3} n_0 m_i U_0^2/2$$

and with phase velocity of order U_0. Since, in the one-dimensional model under consideration, there are no oscillations at an angle to the direction of propagation, the ion-acoustic instability does not appear in the numerical calculation.

The author thanks V. S. Imshennik and R. Z. Sagdeev for their interest in the work and for discussions.

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Received
2 X 1969

CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.