ON THE PROBLEM OF OBSERVING NONLINEAR SYSTEMS

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Abstract

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ON THE PROBLEM OF OBSERVING NONLINEAR SYSTEMS

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The direct solution of the problem of determining the position of a nonlinear system in phase space leads to the necessity of constructing nonlinear resolving operations, the realization of which is achieved by comparatively complicated algorithms ([1]). In a number of cases, however, the computation of the coordinates of a nonlinear system that are not accessible to measurement can be reduced to the procedure of estimating the state of the corresponding linear systems. We shall illustrate this possibility for the case when the motion under study is governed by the differential equation

$$\dot{x}(\tau) = f(\tau, x), \qquad x(\tau) = x_0, \tag{1}$$

where x is an n-dimensional vector of phase coordinates; $f(\tau, x)$ is a nonlinear vector function whose properties guarantee the existence and uniqueness of the solution $x(\tau, \tau_0, x_0)$, and, moreover, in a neighborhood of this solution it has an infinite number of continuous derivatives with respect to x. For simplicity of exposition we shall consider the problem of determining the initial value of the vector $x(\tau_0)$ from the ideal signal

$$y(\tau) = H(\tau)x(\tau), \tag{2}$$

measured on the interval $\tau_0 \le \tau \le t$. We assume here that $y(\tau)$ is an m-dimensional vector; $H(\tau)$ is a continuous $(n \times n)$-matrix and $m < n$.

Instead of (1) let us consider the more general system

$$\dot{z}(\tau, \lambda) = G[\tau, z(\tau, \lambda), \lambda], \qquad z(\tau_0, \lambda) = x_0, \tag{3}$$

depending on the real parameter λ and such that, for $\lambda = 0$, system (3) is linear, while for $\lambda = 1$ it is identical with (1). We shall assume in addition that $G[\tau, z, \lambda]$ is expandable in a series in λ for $|\lambda| \le 1$, and that the radius of convergence of the series representing the solution of equation (3) coincides with the radius of convergence of the series in λ into which the function $G[\tau, z, \lambda]$ is expanded. In what follows we take

$$G[\tau, z(\tau, \lambda), \lambda] = A(\tau) z(\tau, \lambda) + F[\tau, z(\tau, \lambda), \lambda], \tag{4}$$

assuming that $A(\tau)$ is continuous; $F[\tau, z, \lambda]$ has the properties of the function $G[\tau, z, \lambda]$, and with respect to $z(\tau, \lambda)$ satisfies the conditions imposed on $f(\tau, x)$. Under the assumptions made, using the sensitivity functions $\eta^{[\nu]}(\tau) = [\partial^\nu z(\tau, \lambda)/\partial \lambda^\nu]_{\lambda=0}$ of system (3) with respect to λ, we represent $z(\tau, \lambda)$ in the form

$$z(\tau, \lambda) = z(\tau, 0) + \sum_{\nu=1} \frac{\lambda^\nu}{\nu!} \eta^{[\nu]}(\tau).$$

Then, by virtue of the properties of $G[\tau, z, \lambda]$, the integral curve $x(\tau) = z(\tau, 1)$, and the desired vector x_0, turns out to be equal to

$$x_0 = z(\tau_0, 0) + \sum_{\nu=1}^{0} \frac{1}{\nu!} \eta^{[\nu]}(\tau_0). \tag{5}$$

The values $z(\tau_0, 0)$, $\eta^{[\nu]}(\tau_0)$ can be found by successively solving the problem of approximation to $y(\tau)$ by the recursively defined variables

$$y^{[\nu]}(\tau) = y^{[\nu-1]}(\tau) + \frac{1}{\nu!} H(\tau) \eta^{[\nu]}(\tau), \qquad y^{[0]}(\tau) = H(\tau) z(\tau, 0) \tag{6}$$

for $\nu = 0, 1, 2, \ldots$. The functions entering into these equalities satisfy the differential equations

$$\dot{z}(\tau, 0) = A(\tau) z(\tau, 0);$$

$$\dot{\eta}^{[1]}(\tau) = A(\tau) \eta^{[1]}(\tau) + \sum_{q=1}^{n} F_{zq}[\tau, z, 0] \eta_q^{[1]}(\tau) + F_\lambda[\tau, z, 0] \tag{7}$$

and so on, which, in accordance with the sensitivity theory of dynamical systems ([2]), are obtained by differentiating (3) with respect to λ at $\lambda = 0$.

Equations (7) are linear. Therefore, to determine x_0 from (5), one may apply well-developed methods for solving observation problems for linear systems ([3–5]).

To illustrate the rate of convergence of the estimates in accordance with the series (5), we give the results of solving the problem of determining the angular velocities x_1, x_2, x_3 of the rotational motion of a rigid body in the coordinate system $OX_1X_2X_3$ attached to it. In the presence of friction such motion is described by Euler' s equations

$$\dot{x}_i = \sum_{j=1}^{3} a_{ij}x_j + \begin{Vmatrix} b_1x_2x_3 \\ b_2x_1x_3 \\ b_3x_1x_2 \end{Vmatrix}, \qquad i = 1, 2, 3.$$

In the case under consideration one may take

$$G_i[\tau, z(\tau, \lambda), \lambda] := \sum_{j=1}^{3} a_{ij}z_j(\tau, \lambda) + \lambda \begin{Vmatrix} b_1z_2z_3 \\ b_2z_1z_3 \\ b_3z_1z_2 \end{Vmatrix},$$

therefore the system of equations (7) will have the form

$$\dot{z}(\tau, 0) = Az(\tau, 0);$$

$$\dot{\eta}^{[1]}(\tau) = A\eta^{[1]}(\tau) + \begin{Vmatrix} b_1z_2(\tau, 0)z_3(\tau, 0) \\ b_2z_1(\tau, 0)z_3(\tau, 0) \\ b_3z_1(\tau, 0)z_2(\tau, 0) \end{Vmatrix}$$

and so on.

For the case when only $x_1(\tau)$ is measured, and the approximation of the variables (6) to $x_1(\tau)$ is carried out by minimizing quadratic functionals, we give successively improved estimates of the angular velocities as a function of the number of terms of the series (5) used:

ν	x_1, sec^{-1}	x_2, sec^{-1}	x_3, sec^{-1}
0	2.003550	1.504025	0.451852
1	2.003267	1.505508	0.990130
2	2.000998	1.501206	0.996414
3	2.000963	1.500232	0.999615

The calculations correspond to

$$a_{11} = a_{12} = a_{22} = a_{31} = 0, \qquad a_{21} = a_{32} = 1,$$

$$a_{13} = -2, \quad a_{23} = -5, \quad a_{33} = -4, \quad b' = (0.04;\ 0.02;\ 0.03).$$

The true values $x_1(0), x_2(0), x_3(0)$ were taken equal, respectively, to 2.0; 1.50; 1.0 $\sec^{-1}$.

In conclusion, we note that the method presented admits a generalization to the case where the system under study is subject to the action of control functions, and the measuring equation (2) is nonlinear.

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CITED LITERATURE

[1] Ya. N. Roitenberg, DAN, **144**, No. 6, 1225 (1962).
[2] M. L. Bykhovskii, *Fundamentals of the Dynamic Accuracy of Electrical and Mechanical Systems*, Publishing House of the Academy of Sciences of the USSR, 1958.
[3] N. N. Krasovskii, PMM, **28**, no. 1 (1964).
[4] N. N. Krasovskii, *Theory of Motion Control*, "Nauka," 1968.
[5] R. E. Kalman, R. S. Bucy, Trans. ASME, D, **83** (1961).

Note: Figure translations are in progress. See original paper for figures.