ON SOME OSCILLATIONS IN RESONATORS WITH ROUND MIRRORS

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Fig. 1

Figure 1: Fig. 1

Abstract

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ON SOME OSCILLATIONS IN RESONATORS WITH ROUND MIRRORS

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Let us consider a two-dimensional open resonator formed by circular and plane mirrors. Let the radius of curvature of the circular mirror be R, and the distance between the mirrors d. Each ray propagating in the resonator can be characterized by two parameters: the angle φ that this ray makes with the axis of the system, and the dimensionless parameter ρ, equal to the distance from the ray to the center of curvature of the circular mirror, referred to R. Specifying some initial ray and assigning it the index n, one can calculate the parameters of the $(n+1)$-st ray, i.e., the ray arising from the n-th as a result of its reflection from the circular and plane mirrors. As a result of such a calculation we obtain

$$\rho_{n+1} = -\rho_n + t\sin(2\arcsin\rho_n + \varphi_n), \qquad \varphi_{n+1} = -2\arcsin\rho_n - \varphi_n, \quad (1)$$

where $t = 2(1 - d/R)$.

Introduce the notation

$$\rho_n = -p_{n-1}, \qquad q_n = \varphi_n, \quad (2)$$

which simplifies the system of finite-difference equations. We obtain

$$p_{n+1} = -p_n + t\sin q_n, \qquad q_{n+1} = 2\arcsin(-p_n + t\sin q_n) - q_n. \quad (3)$$

Solutions of this system that are conditionally periodic in n describe the proper congruences of rays of the resonator under consideration. System (3) can be linearized near the point $p_n = q_n = 0$. Then the solutions have the form

Fig. 1

$$p_n = A\sqrt[4]{t}\cos\nu_n, \qquad q_n = A(\sqrt[4]{t}\cos\nu_n - \sqrt[4]{2-t}\sin\nu_n), \tag{4}$$

where $\cos\nu = t - 1$.

However, not all solutions of system (3) have a form close to the solutions (4) of the linearized system. In ([1]) it is shown that, for rotations of the general elliptic type, to which the transformation (3) also belongs, between solutions of type (4) there lie nonresonance zones, where the solution has an essentially different character.

Therefore it seemed interesting to obtain solutions of system (3) with the aid of an electronic computer. Figure 1 presents the results of such calculations for $t = 0.52$ and for various initial conditions. The ovals whose center lies at the origin correspond to solutions close to the solutions of the linearized system (4). The small ovals of irregular shape correspond to the so-called nonresonant zones. The precession along the small ovals occurs with frequencies that differ strongly from the frequencies of precession along the ovals with center at the origin, which accounts for their name—the nonresonant zone. Upon reflection of rays in the plane and circular mirrors, zone 0 passes into zone 1, zone 1 into zone 2, and zone 2 into zone 0. Therefore these three zones essentially represent one and the same system of rays. Obviously, the rays corresponding to zones 0, 1, 2 after reflection in the circular or plane mirrors will enter the same self-congruence. In Fig. 1 these rays correspond to zones $0'$, $1'$, $2'$. Thus the number of nonresonant zones is equal to the number of ray streams in Fig. 2.

The choice of the value $t = 0.52$ is due to the following reasons. The magnitude of the nonresonant zone, according to (1), is the larger the smaller is the period in n of that periodic solution near which the nonresonant zone is located. Therefore it was decided to seek the nonresonant zone near a solution with period in n equal to 3. For $t = 0.5$, a solution with period in n equal to 3 already occurs in the linear approximation. Consequently, it could be assumed that for $t = 0.52$ the nonresonant zone lies not too far from the origin and at the same time is sufficiently large. Figure 1 shows that these assumptions were on the whole justified. The nonresonant zones correspond to the resonator oscillations shown in Fig. 2. The phase run from caustic to caustic and back must be equal to $(2N + 1)\pi$, where N is an integer, i.e., the transverse phase condition for the oscillations under consideration has the form

$$k\int \sin\varphi_n \, dx_n = (2N+1)\pi, \tag{5}$$

where k is the wave number and x_n is the coordinate of the point of intersection of the ray and the plane mirror. In accordance with Fig. 2, in condition (6) the index n runs through the values $0, 3, 6, 9, ...$

Fig. 2

Figure 2: Fig. 2

Fig. 2

In view of the conditional periodicity in n, and since

$$x_n = R\left(\frac{p_n}{\cos q_n} - \frac{t}{2}\operatorname{tg} q_n\right), \tag{6}$$

condition (5) can be transformed to the following simple form:

$$kR\int p_n\,dq_n = (2N+1)\pi. \tag{7}$$

Thus, the principal significance in the phase picture (Fig. 1) is the area of the nonresonant zone. In the case considered, this area is approximately 10^{-2}. Consequently, in the optical range ($kR \sim 10^6$, $\lambda \cong 10^{-4}$ cm) the number N can reach a value of 10^4.

As t tends to 1/2, the nonresonant zone will tend to the origin. It may be assumed that the area of the nonresonant zone decreases quadratically with decreasing p and q, i.e., linearly with Δ (see (10)). Thus, the first oscillation of the type under consideration will appear approximately at $q = 3\cdot 10^{-3}$ ($\sim 10'$) and $\Delta \cong 10^{-6}$.

The dependence q, at which the nonresonant zone is located, on t can be estimated as follows. System (3) has a periodic solu-

with period in n equal to 3, for the values of the initial data

$$p_0 = \sin q_0 = \sqrt{\frac{(2t-1)(2t^2+t-2)}{4t(t^2-1)(t-1)^2}}. \tag{8}$$

If t differs little from 1/2, i.e., $t = 1/2 + \Delta$, where Δ is small, then

$$p_0 = \sin q_0 \cong \sqrt{{}^{20}/_{3}\Delta}, \tag{9}$$

and, consequently, $p_0 q_0$ is proportional to Δ. This periodic solution is the center of the nonresonance zone 0. When transformation (3) is applied to it, the center of zone 1 is obtained, and when the same transformation is applied once again, the center of zone 2 is obtained. There are also other periodic solutions with the same period in n, but they are unstable, i.e., they are not centers of nonresonance zones. Such solutions arise, for example, under the initial conditions

$$p_0 = 0, \qquad \cos q_0/2 = 1/2t$$

and they are located at those places where the nonresonance zones come closest to one another. As $t \to 1/2$, these solutions also tend to zero as $\sqrt{\Delta}$.

On the basis of work (3) one may conclude that, in some systems of difference equations less symmetric than system (3), there may exist periodic solutions with period in n equal to 2 and corresponding nonresonance zones. System (3), describing a resonator symmetric with respect to the plane $x = 0$, has no stable solutions with period in n equal to 2.

Knowledge of the ray pattern makes it possible to construct the corresponding wave field [2].

In the three-dimensional case, i.e., with a spherical upper mirror and a plane lower one, analogous oscillations also exist. A representation of these oscillations is not difficult to obtain from the two-dimensional oscillations by rotating Fig. 2 about the axis. The oscillations under consideration are situated closer to the edges of the mirrors than oscillations of the usual type. Therefore, in experimental observation of these oscillations it is necessary, by one means or another, to suppress oscillations of the usual type.

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REFERENCES

1. V. I. Arnold, *Uspekhi Mat. Nauk*, vol. 6 (114), 91 (1963).
2. V. P. Bykov, *Zh. Tekh. Fiz.*, 40, No. 7 (1970).
3. L. B. Luganskii, in: *Electronics of High Powers*, No. 6, Moscow, 1969, p. 130.

Note: Figure translations are in progress. See original paper for figures.