INSTABILITY OF AN INHOMOGENEOUS HIGH-PRESSURE PLASMA IN A HOMOGENEOUS MAGNETIC FIELD

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Abstract

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A. B. MIKHAILOVSKII

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1. Let an inhomogeneous collisionless plasma with pressure $p_0 \gg B_0^2/8\pi$ be in a homogeneous magnetic field B_0, $\partial B_0/\partial x = 0$. (The condition $\partial B_0/\partial x = 0$ is satisfied if $\partial p_0/\partial x = 0$.) Consider plasma perturbations with frequency $\omega \ll (k_z v_{Te}, \omega_{Bi})$ and transverse wavelength $\lambda_\perp \gg \rho_i$ (v_{Te} is the electron thermal velocity, ρ_i and ω_{Bi} are the Larmor radius and cyclotron frequency of the ions, and k_z is the longitudinal wave number). As follows from work ([1]), in the limit of high pressures $\beta \equiv 8\pi p_0/B_0^2 \to \infty$, such perturbations are described by the local dispersion equation

$$\frac{1}{T}\hat{l}_i(TxW) + \frac{i\sqrt{\pi}}{4}\left\{2\hat{l}_i\left(\frac{xW}{T}\right)\hat{l}_i(TxW) - [\hat{l}_i(xW)]^2\right\} = 0. \qquad (1)$$

Here the notation of work ([1]) is used:

$$x = \omega/|k_z|v_{Ti}, \qquad v_{Ti} = (2T/m_i)^{1/2},$$

$$W = e^{-x^2}\left[1 + (2i/\sqrt{\pi})\int_0^x e^{t^2}\,dt\right], \qquad \hat{l}_i = 1 - (\omega_T/\omega)\,\partial/\partial \ln T,$$

$$\omega_T = k_y(\partial T/\partial x)/m_i\omega_{Bi};$$

T is the plasma temperature.

For $x \gg 1$, (1) yields a quadratic equation for ω, so that

$$\omega = {}^2\!/\!_3\,\omega_T(1 \pm i/\sqrt{2}). \qquad (2)$$

It is seen that, with respect to such perturbations, the plasma is unstable. The instability also occurs for $x \ll 1$, and in this case

$$\mathrm{Re}\,\omega = \omega_T/2, \tag{3}$$

$$\gamma \equiv \mathrm{Im}\,\omega = \frac{\pi - 2}{4\sqrt{\pi}} \frac{\omega_T^2}{|k_z| v_{Ti}}. \tag{4}$$

2. If the plasma is described by a system of two-fluid hydrodynamic equations with adiabatic index γ_0 ([2]), then instead of (2) one obtains

$$\omega = \omega_T \frac{\gamma_0 \pm i\sqrt{\gamma_0}}{1 + \gamma_0}. \tag{5}$$

For $\gamma_0 = 2$ this coincides with (2), obtained in the approximation of a collisionless plasma. If the frequency of ion-ion collisions exceeds the oscillation frequency, then $\gamma_0 = 5/2$, and then

$$\omega = {}^5\!/_8\, \omega_T (1 \pm i\sqrt{3/5}). \tag{6}$$

Thus, a collisional plasma with $\beta \gg 1$ and $\nabla p_0 = 0$, like a collisionless one, is unstable.

3. Let us estimate what consequences the instability considered above may lead to. From the pressure-balance equation

$$-\frac{\partial}{\partial x}\left(p + \frac{B^2}{8\pi}\right) = 0 \tag{7}$$

it follows that the perturbed magnetic field B_z' will become of the order of the equilibrium field B_0 when the perturbed pressure p' is such that

$$p'/p_0 \simeq 1/\beta \ll 1. \tag{8}$$

If it is assumed that at large p' the instability is suppressed, then, using the continuity and motion equations and Ohm' s law, one can obtain an estimate for the plasma relaxation time

$$\tau \simeq 4\pi\sigma a^2/c^2, \tag{9}$$

where σ is the conductivity, $a \equiv (\partial \ln T/\partial x)^{-1}$. The quantity τ turns out to be, in order of magnitude, equal to the diffusion time of the magnetic field into the plasma ([3]).

Thus, one may suppose that if no instabilities other than those considered here develop, then over a time of the order of several inverse increments, a small pressure gradient, $\partial \ln p/\partial x \simeq 1/a\beta$, must arise in a plasma with $\nabla p_0 = 0$, and then, over a time of the order of τ, plasma decay will occur.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.