ESTIMATING THE CLOSENESS OF THE STATISTICAL CHARACTERISTICS OF LINEAR NONSTATIONARY AUTOMATIC CONTROL SYSTEMS OF ONE CLASS

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Abstract

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ESTIMATING THE CLOSENESS OF THE STATISTICAL CHARACTERISTICS OF LINEAR NONSTATIONARY AUTOMATIC CONTROL SYSTEMS OF ONE CLASS

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Linear nonstationary automatic control systems (ACS) are considered that consist of a stationary element with a fractional-rational transfer function, closed by feedback with a gain coefficient inversely proportional to a linear function of time. These systems are described by a linear differential equation of Laplace type

$$\sum_{k=0}^{n}(\alpha_k+\beta_k t)y^{(k)}(t)=\sum_{k=0}^{m}\mu_k x^{(k)}(t),$$

where α_k, β_k, μ_k are real constants.

Estimates are obtained for the closeness of the statistical characteristics of the output coordinate of nonstationary ACS, depending on estimates of the closeness of the impulse response functions of their stationary elements. As a result, a number of research methods developed for systems with constant parameters can be transferred to the given class of nonstationary ACS.

Suppose there exists a nonstationary system with impulse response function of the stationary element $\Phi^*(t)$, possessing the desired dynamic characteristics (we shall call it the limiting system). The limiting system transforms an input signal, which is, for example, a nonstationary random process $x(t)$ with mathematical expectation $m_+(t)$ and correlation function $K_+(t_1,t_2)$, into the desired output signal $y^*(t)$, whose mathematical expectation is $m_-^*(t)$ and correlation function $K_-^*(t_1,t_2)$.

Let us consider a method for constructing estimates of the modulus of deviation of the statistical characteristics of the output signal of a nonstationary ACS with impulse response function of the stationary element $\Phi(t)$ from the statistical characteristics of the output of the limiting system, $|m_-(t)-m_-^*(t)|$ and

$|K_-(t_1,t_2)-K_-^*(t_1,t_2)|$, as functions of estimates of the modulus of the difference of the impulse response functions of their stationary elements, for the case when the stationary element of the system is variable. To solve the problem it is necessary to obtain an estimate of the closeness of the weighting functions of the real and limiting nonstationary ACS.

Let the action be applied to the stationary element. The closeness estimates are constructed on the basis of the explicit expression for the solution of the equation describing the process, under zero initial conditions ([1]), which, if the action $x(t)$ is applied to the input of the stationary element and the output variable of the nonstationary ACS is the output of the stationary element, can be written in the form

$$y(t)=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{zt}X(z)T_y(t,z)\,dz, \tag{1}$$

where

$$T_\nu(t,z)=T_\nu(z)e^{-(t-t_0)z}(t-t_0)\int_\infty^z e^{(t-t_0)\eta}\varphi(\eta)\,d\eta \tag{2}$$

is the parametric transfer function, $T_\nu(z)$ is its value for the time instant $t=t_0$; t is the current time; t_0 is the process time; $t\le t_0$.

$$T_\nu(z)=\lim_{t\to t_0}T_\nu(t,z)=\Phi(z)\exp\int_\infty^z\Phi(\eta)\,d\eta,\qquad \varphi(z)=\exp\int_z^\infty\Phi(\eta)\,d\eta,$$

$$\Phi(z)\underset{z\to\infty}{\sim}O(z^{-n}),\qquad n>1.$$

We assume that the system is stable*, i.e., $\Phi(z)$ may have a simple pole at zero, with

$$\lim_{z\to 0}z\Phi(z)>0.$$

The nonzero poles of $\Phi(z)$ lie in the left half-plane.

Taking into account the operational correspondence (2)

$$L^{-1}\left[e^{-(t-t_0)z}(t-t_0)\int_\infty^z e^{(t-t_0)\eta}\varphi(\eta)\,d\eta\right]=\frac{(t_0-t)\varphi(u)}{u-t+t_0},\qquad u\ge 0,\quad t\le t_0,$$

and also (1), (2), we write the solution of the equation in the form of a convolution integral

$$y(t) = \int_0^t x(u) A_\nu(t,u)\,du, \qquad A_\nu(t,u) = \int_0^{t-u} T_\nu(t-u-u_1)\frac{(t_0-t)\varphi(u_1)}{u_1-t+t_0}\,du_1,$$

$$t < t_0, \tag{3}$$

where $A_\nu(t,u)$ is the weighting function of the nonstationary ACS; $T_\nu(t)$ is the section of the weighting function for the time instant $t = t_0$.

Expressing the functions $T_\nu(t)$ and $\varphi(t)$ in terms of their values corresponding to the limiting system, $T_\nu(t) = T^*_\nu(t) + \Delta T_\nu(t)$, $\varphi(t) = \varphi^*(t) + \Delta\varphi(t)$, and substituting into (3), we obtain

$$|A_\nu(t,u) - A^*_\nu(t,u)| \le \int_0^{t-u} |T^*_\nu(t-u-u_1)|\,\frac{(t_0-t)|\Delta\varphi(u_1)|}{u_1-t+t_0}\,du_1 +$$

$$+ \int_0^{t-u} |\Delta T_\nu(t-u-u_1)|\,\frac{(t_0-t)\left[|\varphi^*(u_1)| + |\Delta\varphi(u_1)|\right]}{u_1-t+t_0}\,du_1. \tag{4}$$

We express the estimate of the modulus of the difference between the sections of the weighting functions of the real and limiting systems for $t = t_0$, $\Delta T_\nu(t)$, and $\Delta\varphi(t)$ in terms of the estimate of the deviation of the impulse response function of the stationary element of the real system from the impulse response function of the stationary element of the limiting system. $\Delta T_\nu(t)$ may be written as follows:

$$\Delta T_\nu(t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{zt} T^*_\nu(z)\left[\exp\int_z^\infty \Delta\Phi(\eta)\,d\eta - 1\right] dz -$$

$$-\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{zt} T^*_\xi(z)\Delta\Phi(z)\exp\int_z^\infty \Delta\Phi(\eta)\,d\eta\,dz;$$

here

$$\Delta\Phi(z) = \Phi^*(z) - \Phi(z), \qquad T^*_\xi(z) = \exp\int_\infty^z \Phi^*(\eta)\,d\eta.$$

* The stability condition was formulated by A. T. Barabanov.

Expanding the function $\exp \int_z^\infty \Delta\Phi(\eta)d\eta$ in a series and applying the convolution operation, we obtain

$$|\Delta \dot{T}_\nu(t)| \leq \int_0^t |T_\nu^*(\tau)| \left| \sum_{k=1}^\infty \frac{1}{k!} \Lambda_k(t-\tau) \right| d\tau + \int_0^t |T_\varepsilon^*(\tau)| \left| l_0(t-\tau) + \sum_{k=1}^\infty \frac{1}{k!} l_k(t-\tau) \right| d\tau, \tag{5}$$

$$l_k(t) = \int_0^t \Delta\Phi(\tau)\Lambda_k(t-\tau)d\tau, \quad \Lambda_0(t) = \delta(t) \text{—delta function}, \quad \Lambda_1(t) = \frac{\Delta\Phi(t)}{t},$$

$$\Lambda_k(t) = \int_0^t \Lambda_1(\tau)\Lambda_{k-1}(t-\tau)d\tau.$$

The estimate of the modulus of the difference between the impulse transition functions of the stationary links of the real and limiting ACS can be written in the form $|\Delta\Phi(t)| < \varepsilon_0\varepsilon(t)$, $\varepsilon_0 = \text{const}$, where $\varepsilon(t)$ is a certain function of time satisfying, in the case under consideration, the condition for the existence of the integral $\int_z^\infty \varepsilon(\eta)d\eta$, i.e. $\varepsilon(z) \underset{z\to\infty}{\sim} O(z^{-\nu})$, $\nu > 1$. In this case we have

$$\Lambda_k(t) < \varepsilon_0^k \Lambda_k^*(t), \quad l_k(t) < \varepsilon_0^{k+1} l_k^*(t), \quad \text{where } \Lambda_1^*(t) = \frac{\varepsilon(t)}{t}, \quad \Lambda_k^*(t) \int_0^t \Lambda_1^*(\tau)\Lambda_{k-1}^*$$

$$\times (t-\tau)d\tau, \quad l_k^*(t) = \int_0^t \varepsilon(\tau)\Lambda_k^*(t-\tau)d\tau.$$

Taking formula (5) into account, we obtain an estimate of the closeness of the sections of the weighting functions of the real and limiting ACS for the time instant $t = t_0$

$$|\Delta T_\nu(t)| < \Delta_{T_\nu}(t), \quad \Delta_{T_\nu}(t) = \int_0^t |T_\nu^*(\tau)|\Lambda^*(t-\tau)d\tau +$$

$$+ \int_0^t |T_\varepsilon^*(\tau)|L^*(t-\tau)d\tau, \tag{6}$$

where

$$\Lambda^*(t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{zt}\left[\exp \varepsilon_0 \int_z^\infty \varepsilon(\eta)d\eta - 1\right]dz,$$

$$L^*(t) = \frac{\varepsilon_0}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{zt}\varepsilon(z)\exp \varepsilon_0 \int_z^\infty \varepsilon(\eta)d\eta\, dz.$$

The estimate of the modulus $\Delta\varphi(t)$ can be found in the form

$$\Delta_\varphi(t) = \int_0^t |\varphi^*(\tau)|\Lambda^*(t-\tau)d\tau. \tag{7}$$

As a result, according to (4), (6), (7), the estimate of the closeness of the weighting functions of the real and limiting nonstationary ACS is determined as follows:

$$|A_\nu(t,u) - A_\nu^*(t,u)| < \Delta_{A_\nu}(t,u),$$

$$\Delta_{A_\nu}(t,u) = \int_0^{t-u} |T_\nu^*(t-u-u_1)|\,\frac{(t_0-t)\Delta_\varphi(u_1)}{u_1-t+t_0}\,du_1+$$

$$+\int_0^{t-u} \Delta_{T_\nu}(t-u-u_1)\frac{(t_0-t)\left[|\varphi^*(u_1)| + \Delta_\varphi(u_1)\right]}{u_1-t+t_0}\,du_1.$$

It is now easy to obtain estimates of the closeness of the statistical characteristics of the output of the systems under consideration. Thus, the estimate of the closeness of the mathematical expectations of the real and limiting automatic control systems has the form

$$|m_-(t) - m_-^*(t)| < \Delta_m(t)^*, \qquad \Delta_m(t) = \int_0^t |m_+(u)|\,\Delta_{A_\nu}(t,u)\,du.$$

Taking into account the known relation between the correlation functions of the input and output of a nonstationary automatic control system,

$$K_-(t_1,t_2) = \int_0^{t_1} A_\nu(t_1,v)\,dv\int_0^{t_2} A_\nu(t_2,u)K_+(u,v)\,du, \qquad t_1,t_2 < t_0,$$

an estimate of the closeness of the correlation functions of the output signal of the real and limiting automatic control systems can be obtained in the form

$$|K_-(t_1,t_2) - K_-^*(t_1,t_2)| < \Delta_k(t_1,t_2),$$

$$\Delta_k(t_1,t_2) = \int_0^{t_1} |A_\nu^*(t_1,v)|\,dv \int_0^{t_2} \Delta_{A_\nu}(t_2,u)\,|K_+(u,v)|\,du+$$

$$+\int_0^{t_1} \Delta_{A_\nu}(t_1,v)\,dv \int_0^{t_2} |A_\nu^*(t_2,u)|\,|K_+(u,v)|\,du+$$

$$+\int_0^{t_1} \Delta_{A_\nu}(t_1,v)\,dv \int_0^{t_2} \Delta_{A_\nu}(t_2,u)\,|K_+(u,v)|\,du.$$

Thus, by means of the method set forth above, it proves possible to construct estimates of the closeness of the statistical characteristics of nonstationary automatic control systems as functions of estimates of the closeness of their stationary elements. This circumstance makes it possible to transfer to the class of nonstationary automatic control systems under consideration a number of investigation methods well developed for systems with constant parameters. Thus, for example, an estimate of the closeness of stationary elements can characterize the deviation of the frequency characteristics of the stationary element of the real system from the frequency characteristics of the stationary element of the limiting system ([3]). To the given class of nonstationary automatic control systems there may be transferred the method of synthesis of automatic control systems with constant parameters on the basis of the transition functions of A. A. Krasovskii ([4]), the method of investigation of automatic control systems on the basis of generalized integral quadratic criteria ([5]), and others.

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* The estimate $\Delta_m(t)$ for the time instant $t = t_0$ was obtained earlier by the author and A. T. Barabanov in the general case in the form of series.

Note: Figure translations are in progress. See original paper for figures.