Geophysics

A. S. Monin, V. T. Neiman, B. N. Filyushkin

1970

SovietRxiv

Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

Geophysics

A. S. Monin, V. T. Neiman, B. N. Filyushkin

On Density Stratification in the Ocean

(Presented by Academician L. I. Sedov, 10 IX 1969)

Despite the low compressibility of seawater, its density ρ in the ocean varies with depth z—in the overwhelming majority of cases it increases, so that the ocean is an important geophysical example of a stably stratified fluid. The increase of ρ from the surface to the ocean bottom in the sharpest cases—in equatorial waters —reaches 0.5-0.6% (see, for example, ([1]), Table V), with the main part of the increase in water density occurring in the so-called jump layer, located at depths of the order of 100 m and capable of serving as a "liquid bottom" for submarines. In this layer the curve $\rho(z)$ has an inflection point. Below it the density increases with depth very slowly, and in order to elucidate the regularities of this increase, instead of $\rho(z)$ it is more convenient to consider the more sensitive characteristic $d\rho/dz$, or the Brunt-Väisälä frequency N expressed through it, defined by the relation (see, for example, ([2]))

$$N^2 = \frac{g}{\rho}\left[\frac{\partial\rho}{\partial z} - \left(\frac{\partial\rho}{\partial z}\right)_\eta\right], \tag{1}$$

where g is the acceleration due to gravity and $(\partial\rho/\partial z)_\eta$ is the vertical derivative at constant entropy η. N has the meaning of the frequency of inertial oscillations (in the field of restoring Archimedean forces) of a fluid particle adiabatically displaced vertically from its equilibrium level. The frequency N generally increases with depth from the ocean surface to the jump layer, where it reaches a maximum (corresponding to an oscillation period $T = 2\pi/N$ of the order of 10 min), and from the jump layer to the bottom it generally decreases (by several tens of times), so that below the jump layer the stability of the stratification weakens with depth.

Fig. 1

For the analysis of the stratification curves $N(z)$ we used data from 40 hydrographic stations in the northern half of the Pacific Ocean (mainly from the vessel *Vityaz*), as well as from several stations of the vessels *Galathea*, *Snellius*, *Horizon*, and *Alexander Agassiz*, and computed N, for simplicity, by the approximate formula

$$N^2 \approx \frac{g}{\rho}\frac{\partial \sigma_t}{\partial z}, \tag{2}$$

where σ_t is the so-called conditional density, i.e., the deviation of ρ from the standard value 1 g/cm^3, computed from the temperature and salinity of the water without taking into account the correction for hydrostatic pressure. The obtained curves $N(z)$ in the upper layer of the ocean have a complex and varied structure, often with several extrema, but in the interior layers of the ocean, at depths of 500–5000 m, they are fairly well described by the simple law of distance from the surface

$$Nz = u_* = \text{const}, \tag{3}$$

i.e., the frequency N turns out to be inversely proportional to the depth z.

As an example, Fig. 1 gives, in the coordinates $(\ln N, \ln z)$, the stratification curve for the *Vityaz* station No. 4311 ($\varphi = 20°03'$ N, $\lambda = 151°49'$ W, ocean depth $H = 5293$ m).

Moreover, law (3) proves to be universal in the sense that the constant u_* for different stations is approximately the same (it is equal to $u_* \approx 2.2$ m/sec).

Fig. 2 gives the values of Nz at all depths z for the 40 hydrographic stations considered by us (the vertical segments indicate the standard deviations from the mean value at each depth). These values vary within the limits 1–4 m/sec, and the variability of the values of Nz is produced mainly by the scatter of individual points, and not by displacements of the stratification curves as a whole relative to the universal curve (3). The scatter may be produced by random errors in measurements of temperature and salinity, by the inaccuracy of the approximate formula (2), and sometimes by the presence in the ocean of internal surfaces of separation between water masses of different origin. In comparison with the full variability of N (2 orders of magnitude), this scatter may be regarded as small.

In the near-bottom layer of the ocean, below the region of action of law (3), the curves $N(z)$ lose their universal form. Sometimes N decreases there with depth more rapidly than according to law (3) (as should occur, in particular, in stagnant basins with near-bottom convection produced by the geothermal heat flux), and sometimes more slowly (for example, when cold waters of Antarctic origin are present in the near-bottom layer).

To interpret law (3), one may propose the hypothesis of universality of the statistical distribution, in the interior layers of the ocean, of turbulent patches

Fig. 2

Figure 2: Fig. 2

(“pancakes”) formed there when internal waves lose stability and producing vertical exchange. If the vertical flux of momentum

$$\tau = \rho u_*^2$$

created by turbulent patches (u_* is a friction velocity of order 1 cm/sec) is approximately the same everywhere, then law (3) can be rewritten in the form $T = Cz/u_*$ with a numerical factor C of order 0.1.

Fig. 2

For a more consistent use of the theory of similarity for turbulence in a stratified fluid (set forth in [3], Chap. IV), it is more convenient to consider the ocean as a flow near a solid wall (the bottom) and, instead of the depth z, to use the height above the bottom $h = H - z$ (H is the total depth of the ocean). It follows from the theory that in the case of stable stratification, for large h, the typical scales of turbulent inhomogene-

···have the order

$$L = u_*^3 (gM/\rho)^{-1}, \tag{4}$$

where $M = \overline{\rho' w'}$ is the vertical turbulent flux of mass (w is the vertical velocity, the prime denotes fluctuations, and the bar statistical averaging). The velocity gradient $\partial u/\partial h$, for large h, asymptotically approaches a constant of order u_*/L, while the gradients of temperature and salinity (which determine the density gradient) must increase with height as $1/\alpha(h)$, where α is the ratio of the exchange coefficients for heat (and salt) and for momentum. The transfer of inhomogeneities of size L with velocity $h\,\partial u/\partial h$ creates a local frequency

$$N = \frac{h\,\partial u/\partial h}{L} \left(= Ah; \quad A = \frac{1}{L}\frac{\partial u}{\partial h} \sim \frac{u_*}{L^2} \right), \tag{5}$$

Following R. Long, who kindly sent us a preprint of his article ([4]), one may formulate a resonance condition ensuring the propagation of internal waves throughout the entire thickness of the ocean (this condition corresponds to $\alpha(h) \sim h^{-2}$). Our check of the law of distance from the bottom, $N = Ah$, showed that it is satisfactorily fulfilled for most of the above-mentioned hydrological stations at depths below 1-2 km.

As an example, Fig. 3 gives, in the coordinates $(N/h,\ h)$, the stratification curve for station “Vityaz” No. 4371 ($\varphi = 27°06'9$ N, $\lambda = 153°45'$ W, $H = 6020$

Fig. 3

Figure 3: Fig. 3

m). From what has been said above it is clear that law (5) is not universal: the constant A proves to be different for different stations (it varies within the limits $(1–9) \cdot 10^{-7}$ m$^{-1} \cdot$ sec^{-1}).

Fig. 3

Combining laws (3) and (5), we obtain $A = w_*/z(H - z)$. This quantity is close to constant (depends little on z) in the layer near the maximum of the function $z(H - z)$, which is reached at $z = H/2$. Thus, in the middle of the ocean thickness both laws (3) and (5) may operate.

P. P. Shirshov Institute of OceanologyAcademy of Sciences of the USSRMoscow

Received29 VIII 1969

REFERENCES CITED

1. *Pacific Ocean*, 2, "Nauka," 1968.

2. K. Eckart, *Hydrodynamics of Oceans and Atmospheres*, IL, 1963.

3. A. S. Monin, A. M. Yaglom, *Statistical Hydromechanics*, Part 1, "Nauka," 1965.

4. R. R. Long, *J. Fluid Mech.* (1969).

Note: Figure translations are in progress. See original paper for figures.