ON THE LIMITING DEFLECTION OF A VISCOELASTIC BEAM

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Abstract

Full Text

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THEORY OF ELASTICITY

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ON THE LIMITING DEFLECTION OF A VISCOELASTIC BEAM

(Presented by Academician Yu. N. Rabotnov on 18 V 1970)

1°. We shall consider a viscoelastic beam of variable cross-section and finite length l, lying on an elastic foundation of Winkler type.

As is known (^{5,8}), the deflection $y(x,t)$ of its axis is described by the following boundary-value problem:

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 y}{\partial x^2}\right) + Cy(x,t) + C\int_0^t K(t,\tau)y(x,\tau)\,d\tau =$$

$$= p(x,t) + \int_0^t K(t,\tau)p(x,\tau)\,d\tau; \qquad 0 \le x \le l; \quad 0 \le t < \infty, \tag{1}$$

$$U_i[y] = 0, \tag{2}$$

where (2) are the boundary conditions describing the nature of the fastening of the beam at its ends, $p(x,t)$ is the external transverse load, $K(t,\tau)$ is the kernel characterizing the hereditary properties of the material of the beam, $I(x)$ is the moment of inertia of the beam cross-section relative to the axis, C is the foundation coefficient, and E is the instantaneous modulus of elasticity. We shall assume that E and C are constants.

Let $\alpha(t)$ be some continuous function, positive for $t \ge 0$. We shall assume that

a)

$$\sup_{0\le t<\infty}\int_0^t |K(t,\tau)|\frac{\alpha(t)}{\alpha(\tau)}\,d\tau < \infty;$$

b)

$$\lim_{t\to\infty}\int_E K(t,\tau)\frac{\alpha(t)}{\alpha(\tau)}\,d\tau = 0$$

for any measurable bounded set $E \subset [0,\infty)$;

c)

$$l_K = \lim_{t\to\infty} \int_0^t K(t,\tau)\frac{\alpha(t)}{\alpha(\tau)}\,d\tau$$

exists.

Let us note that if $K(t,\tau) = K_0(t-\tau)$ and

$$\alpha(t) = e^{-\theta t}, \tag{3}$$

where θ is some real number, then conditions a), b), c) are equivalent to the following:

$$\int_0^\infty |K_1(t)|e^{-\theta t}dt < \infty. \tag{4}$$

We are interested in the conditions under which the existence of the limit, uniform in x,

$$(\mathrm{L}p)(x) = \lim_{t\to\infty} p(x,t)\alpha(t) \tag{5}$$

implies the existence of the limit, uniform in x,

$$(\mathrm{L}y)(x) = \lim_{t\to\infty} y(x,t)\alpha(t) \tag{5”}$$

and how to find this latter limit.

In particular, we are interested in the case when the equality

$$(Lp)(x) \equiv 0 \tag{6}$$

implies the equality

$$(Ly)(x) \equiv 0. \tag{6”}$$

2°. Let us denote by $Q(x,\xi)$ the Green’ s function of the differential operator Λ_x:

$$\Lambda_x[y] = \frac{d^2}{dx^2}\left(I(x)\frac{d^2y}{dx^2}\right) + \frac{C}{E}y$$

under the boundary conditions (2), and by Q the Fredholm operator acting in the Banach space $C[0,l]$:

$$(Qg)(x) = \int_0^l Q(x,\xi)g(\xi)\,d\xi. \tag{7}$$

With the aid of Tauberian theorems of the type of Paley–Wiener–Gelfand ($^{1-4}$), the following propositions can be proved.

Theorem 1. If

$$|E/Cq_0| > \lim_{s\to\infty} \sup_{s\le t<\infty} \int_s^t |K(t,\tau)| \frac{\alpha(t)}{\alpha(\tau)}\,d\tau, \tag{8}$$

where q_0 is the eigenvalue of the operator Q largest in modulus, then the existence of the limit (5) implies the existence of (5″). Moreover,

$$(Ly)(x) = \frac{1}{C}\left(l_{kQ} + \frac{E}{C}I\right)^{-1} Q(Lp + LVp)(x). \tag{9}$$

Condition (8) is also sufficient for (6) to imply (6″).

Theorem 2. Let $\alpha(t)$ have the form (3) and

$$K(t,\tau) = K_0(t-\tau) + \widetilde{K}(t,\tau), \tag{10}$$

where $K_0(t)$ satisfies condition (4), and $\widetilde{K}(t,\tau)$ satisfies conditions a), b), c) and also the condition

$$\lim_{s\to\infty} \sup_{s\le t<\infty} \int_s^t |\widetilde{K}(t,\tau)| e^{-\theta(t-\tau)}\,d\tau = 0. \tag{11}$$

Then the condition

$$-\frac{E}{C} \ne q_i k_0(W); \qquad \operatorname{Re} W \ge 0; \qquad q_i \in \sigma(Q), \tag{12}$$

where $k_0(W)$ is the Laplace transform of the function $K_0(t)$, and $\sigma(Q)$ is the spectrum of the operator Q (7) in $C[0,l]$, is sufficient for the existence of the limit (5) to imply the existence of the limit (5″). In this case formula (9) holds.

If

$$k_0(W) + 1 \ne 0; \qquad \operatorname{Re} W \ge 0, \tag{13}$$

then condition (12) is not only sufficient but also necessary for this.

Theorem 3. Let

1) $\alpha(0) = 1; \quad \alpha(t+\tau) \leq \alpha(t) \cdot \alpha(\tau); \quad \theta = \lim_{t\to\infty} \frac{\ln \alpha(t)}{-t} < \infty;$
2) $K(t,\tau)$ be representable in the form (10);
3) $\widetilde{K}(t,\tau)$ satisfy conditions a), b), c) and

$$\lim_{s\to\infty} \sup_{t<t\leq\infty} \int_s^t |\widetilde{K}(t,\tau)| \frac{\alpha(t)}{\alpha(\tau)}\, d\tau = 0;$$

4)

$$\int_0^\infty |K_0(t)|\, \alpha(t)\, dt < \infty.$$

Then condition (12) is sufficient for (6) to imply (6″).

Analogous results are obtained when the problem of buckling of a viscoelastic rod is investigated by our method [6].

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Note: Figure translations are in progress. See original paper for figures.