ON THE EXISTENCE OF INTERNAL SHOCK WAVES IN THE FLOW OF GAS PAST BLUNTED CONES

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Abstract

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ON THE EXISTENCE OF INTERNAL SHOCK WAVES IN THE FLOW OF GAS PAST BLUNTED CONES

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In a number of works, assumptions have been made concerning the possibility of the occurrence, in the field of a supersonic flow of an ideal gas behind the bow shock wave formed by a cone with a spherical blunt nose, of internal shock waves or, as they are more often called, "hanging discontinuities of compaction." In ([1]) attention was first drawn to such a possibility: in some cases, when there is very sharp deceleration of the flow in passing from the circular part of the contour to the rectilinear part, two neighboring characteristics of the first family issuing from this place intersect. This fact indicates the appearance in the flow field of a new "hanging discontinuity." In ([2]) it was assumed that internal shock waves exist in those cases when the gas flow near the surface of the cone is overexpanded (the pressure on some portion of the surface downstream from the junction point is less than the asymptotic value). In ([3]), on the basis of experiments, the occurrence of an internal shock wave near a cone with spherical blunting at a Mach number of the undisturbed flow $M_\infty = 6.1$ and cone half-angle $\beta_k = 15^\circ$ was also assumed. Other assumptions were also made; however, the range of values of the parameters for which internal shock waves arise was not established. Moreover, no specific examples were given of flows in which internal shock waves occur near blunt cones. We note that, near blunt cones with a break in the generators at the junction point, the existence of internal shock waves has been found in a number of theoretical and experimental works (see, for example, ([4])).

In the present communication the following is shown. In the field of an axisymmetric flow of a perfect gas behind the bow shock wave formed by a cone with spherical blunting, without a break of the generators at the junction point, internal shock waves can arise. Their existence depends on the values of the parameters M_∞ and β_k. For $1.25 \leq M_\infty \leq 2$, internal shock waves were found for all $\beta_k < 35^\circ$ (so long as supersonic flow near the cone is possible). For $M_\infty \geq 6$ they were not found for $\beta_k \leq 40^\circ$. For $M_\infty = 4$, internal shock waves were found for $\beta_k \geq 25^\circ$ and were not found for $\beta_k \leq 20^\circ$.

The investigation on the basis of which these conclusions were drawn was carried out by finite-difference methods. First, by establishing the solution in time in solving the nonstationary problem, the steady flow near a sphere was determined; for this purpose the method of ([5]) was used. Then, from the initial data obtained, the supersonic flow was calculated by the method of ([6]). Further, from the values of the functions determined with high accuracy on a sufficiently dense grid, the field of characteristics was constructed.

The numerical methods used for the investigation make it possible to calculate flows with internal shock waves of not very large intensity. In this case the internal shock waves are well localized by the appearance, in the gas-dynamic functions, of oscillations of a specific character in a narrow region of the flow. In the region where the internal ⋯

shock wave, the characteristics of one family coalesce. Thus, from the field of characteristics one can establish the existence and position of internal shock waves. The numerical values used for constructing the characteristics were determined with an accuracy of not less than 0.1%. Therefore, from the numerical solution obtained, internal shock waves can be detected if they have an intensity of at least $(1 \div 0.5)\%$, i.e., a pressure jump $p_2/p_1 = 1.01 \div 1.005$.

Twenty variants of flows were investigated, differing from one another in the values of the parameters M_∞ and β_k. In Table 1, plus signs mark variants with internal shock waves, and minus signs those without them. The letter H marks the values of the parameters M_∞, β_k for which a supersonic conical flow is impossible. For the remaining pairs M_∞, β_k, the investigation was not carried out (blank cells).

Table 1

M_∞	β_k, deg.	β_k, deg.	β_k, deg.	β_k, deg.	β_k, deg.	β_k, deg.	β_k, deg.	β_k, deg.	β_k, deg.
	0	5	10	15	20	25	30	35	40
1.25	+	+			H	H	H	H	H
1.5	+	+		+			H	H	H
2	+	+	+	+	+	+	+		H
4					−	+			
6								−	
8				−		−			
20			−			−			

In Figs. 1–3, in the cylindrical coordinate system (z, r), in one meridional plane, some characteristics of the first family are shown for three flow variants. The shock waves and the generators of the cones are shown by heavy lines. All linear dimensions are referred to the radius of the spherical bluntness.

Fig. 2

Figure 1: Fig. 2

Fig. 3

Figure 2: Fig. 3

Figure 1 presents the variant with $M_\infty = 1.25$, $\beta_k = 0^\circ$. The first two characteristics emerge from points on the surface of the sphere; the third characteristic emerges from the junction point $z_0 = 1.0$; the remaining characteristics emerge from points on the surface of the cylinder. It is clearly seen how the characteristics, converging, form an internal shock wave.

Fig. 1

In all the investigated cases of flows around blunt cylinders, the internal shock waves that formed do not overtake the bow shock wave and, like it, as $z \to \infty$ degenerate into a Mach wave.* The internal shock wave around a cylinder is at first slightly concave to-

* The calculations were carried out up to $z = (5 \div 10) \cdot 10^3$.

toward the oncoming flow, and then convex. Its intensity is not great, and at the point of inflection the intensity of the internal shock wave probably has a maximum.

Figure 2 gives a variant of the flow for $M_\infty = 2$, $\beta_k = 20^\circ$. The first characteristic emerges from the junction point $z_0 = 0.6580$, and the others from points on the cone surface. They form an internal shock wave, which catches up with the bow shock wave and interacts with it. As a result of the interaction, the inclination of the bow shock wave increases. In addition, at the point of interaction a compression wave and a tangential discontinuity are formed. The intensity of the internal shock wave increases in the direction of the point of interaction. In the vicinity of this point, in the present case, its intensity is comparable with the intensity of the bow shock wave. Thus, the pressure jump at the bow shock wave is $p_1/p_\infty = 1.37$, and at the internal shock wave $p_2/p_1 = 1.11$. In this case (and in other cases at larger values of β_k) the internal shock wave has a shape convex toward the oncoming flow.

Fig. 2

Fig. 3

Figure 3 shows a case in which no internal shock wave is formed. In this variant $M_\infty = 6$, $\beta_k = 35^\circ$. The first two characteristics emerge from points on the sphere surface, the third from the junction point $z_0 = 0.4264$, and the others from points on the cone surface.

In all the variants listed in Table 1, the magnitude of the velocity vector behind the internal shock waves is greater than the local speed of sound. On the cone surface the magnitude of the velocity vector V is also greater than the local speed of sound, with $V(z)$ a decreasing function downstream from the junction point in those cases where internal shock waves are found (up to the junction point $V(z)$ is an increasing function). In those flows in which no internal shock waves are found, V downstream does not decrease immediately from the junction point. The pressure on the cone surface has the opposite law of variation.

Let us note the analogy between the flow considered here and the flow over a convex profile. In [7] it is shown that potential flow in a local supersonic zone breaks down if a convex profile is straightened. In that case a shock wave is formed. In the same work a theorem is proved: on a rectilinear portion of a profile in a supersonic zone the velocity decreases. In [8] a similar result was obtained for the case of nonpotential symmetric flow past a smooth convex profile.

with a straight segment. In the present note it is shown that, when axisymmetric bodies composed of a sphere and truncated cones, without a break in the generatrix at the point of junction, are flowed around by a nonpotential flow, in a number of cases a continuous transition from the flow near the sphere to the flow near the cone is impossible. In such cases an internal shock wave arises. In a number of other cases a continuous transition is possible.

On the basis of the investigation carried out, one may suppose that internal shock waves exist in supersonic flow for all β_k and for $1 \leq M_\infty < 1.25$. However, in order to give an exact answer to this question, and also to determine in more detail the limits of variation of the parameters M_∞, β_k for which internal shock waves exist when $2 < M_\infty < 6$, it is necessary to carry out additional investigations.

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Note: Figure translations are in progress. See original paper for figures.