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Abstract

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MATHEMATICAL PHYSICS

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FLUX OF SINGLY SCATTERED NEUTRONS FROM A POINT SOURCE IN AN INFINITE MEDIUM

(Presented by Academician A. N. Tikhonov on 6 II 1970)

In the numerical solution of problems in the theory of neutron slowing-down (by difference methods or by the Monte Carlo method), it is sometimes necessary to isolate analytically the flux of singly scattered neutrons, since the solution of the transport equation may have singularities. In the present work, with the aid of the fundamental solution of the transport operator, an analytic expression is obtained for the flux of singly scattered neutrons from a monoenergetic point source in a homogeneous infinite medium, as well as asymptotic formulas for the numerical calculation of functionals of this flux. A closely related problem, from a somewhat different point of view, was considered in ([1]).

The singularities of the solution of the transport equation are determined by the first iterations (with respect to collisions). For the one-speed transport equation, the singularities of the solution were studied in ([2,3]).

Consider the transport equation for an elastically scattering medium with a point isotropic monoenergetic source

$$\Omega \,\mathrm{grad}\, f(\mathbf{x},\Omega,E) + \Sigma(E) f(\mathbf{x},\Omega,E) =$$

$$= \int \Sigma_s(E' \to E, \Omega\Omega') f(\mathbf{x},\Omega',E')\, dE'\, d\Omega' + \frac{1}{4\pi}\delta(\mathbf{x})\delta(E-E_0). \qquad (1)$$

Here $f(\mathbf{x},\Omega,E)$ is the neutron flux, $\mathbf{x}$ is the spatial variable, Ω is the unit velocity vector, E is the energy, $\Sigma(E)$ is the total cross section for the interaction of neutrons with matter, $\Sigma_s(E' \to E, \Omega\Omega')$ is the differential cross section for elastic scattering, as a result of which a neutron with energy E' and flight direction Ω' acquires energy E and flight direction Ω,

Fig. 1

Figure 1: Fig. 1

$$\Sigma_s(E' \to E, \mu) = \frac{\Sigma_s(E') \left(\sqrt{A^2 - 1 + \mu^2} + \mu\right)^2}{4\pi A \sqrt{A^2 - 1 + \mu^2}} \, \delta \left(E - E_0 \left(\frac{\sqrt{A^2 - 1 + \mu^2} + \mu}{A + 1} \right)^2 \right), \tag{2}$$

where μ is the cosine of the scattering angle, $\Sigma_s(E')$ is the scattering cross section, and A is the mass of the scattering nucleus.

The fundamental solution of the transport operator appearing in the left-hand side of equation (1) has the form

$$\varepsilon(\mathbf{x}, \Omega, E) = \frac{1}{|\mathbf{x}|^2} \exp[-\Sigma(E)|\mathbf{x}|] \, \delta \left(\Omega - \frac{\mathbf{x}}{|\mathbf{x}|} \right)$$

(see ([4])).

Let us compute the flux f_0 of neutrons that have not undergone a single collision:

$$f_0(\mathbf{x}, \Omega, E) = \varepsilon(\mathbf{x}, \Omega, E) * \frac{\delta(\mathbf{x})\delta(E - E_0)}{4\pi} =$$

$$= \frac{\delta(E - E_0)\delta(\Omega - \mathbf{x}/|\mathbf{x}|)}{4\pi|\mathbf{x}|^2} \exp[-\Sigma(E)|\mathbf{x}|].$$

Fig. 1

Here and below the convolutions are performed only with respect to the spatial variables. The flux of singly scattered neutrons $f_1(\mathbf{x}, \ , E)$ is calculated by the formula

$$f_1(\mathbf{x}, \ , E) = \varepsilon(\mathbf{x}, \ , E) * F_0(\mathbf{x}, \ , E), \tag{3}$$

where

$$F_0(\mathbf{x}, \ , E) = \int \Sigma_s(E' \to E, \ ') \, f_0(\mathbf{x}, \ ', E') \, dE' \, d\Omega'$$
$$= \frac{\Sigma_s(E_0 \to E, \mathbf{x} \ /|\mathbf{x}|)}{4\pi|\mathbf{x}|^2} \exp[-\Sigma(E_0)|\mathbf{x}|].$$

In computing the convolution (3), we pass to spherical coordinates ($\xi = s$):

$$f_1(\mathbf{x},\ ,E) = \int d\xi\, \frac{\exp[-\Sigma(E)|\xi|]}{|\xi|^2}\delta\Big(\ -\frac{\xi}{|\xi|}\Big)\, \frac{\Sigma_s\big(E_0\to E, \frac{\mathbf{x}-\xi}{|\mathbf{x}-\xi|}\ \big)}{4\pi|\mathbf{x}-\xi|^2}$$
$$\times \exp[-\Sigma(E_0)|\mathbf{x}-\xi|]$$
$$= \int_0^\infty ds\, \frac{\Sigma_s\big(E_0\to E, \frac{\mathbf{x}-s}{|\mathbf{x}-s\ |}\ \big)}{4\pi|\mathbf{x}-s\ |^2}\exp[-\Sigma(E)s - \Sigma(E_0)|\mathbf{x}-s\ |].$$

We use expression (2) and the formula

$$\int_0^\infty ds\, \Phi(s)\, \delta(E-\varphi(s)) = \frac{\theta(E-\varphi_{\min})-\theta(E-\varphi_{\max})}{|\partial\varphi/\partial s|_{s=g(E)}}\, \Phi(g(E)),$$

where $\varphi_{\min} = \min\{\varphi(0),\varphi(\infty)\}$, $\varphi_{\max} = \max\{\varphi(0),\varphi(\infty)\}$, $\theta(s) = (0$ for $s<0$, 1 for $s>0)$, and $g(E)$ is the function inverse to $\varphi(s)$. We obtain

$$f_1(\mathbf{x},\ ,E) \equiv f_1(r,\mu,E) =$$

$$= \frac{\Big[\theta\Big(E - E_0\left(\frac{A-1}{A+1}\right)^2\Big) - \theta\Big(E-E_0\left(\frac{\sqrt{A^2-1+\mu^2}+\mu}{A+1}\right)^2\Big)\Big]\,\Sigma_s(E_0)(A+1)^2}{(4\pi)^2\, 2AE_0 r\,\sqrt{1-\alpha^2(E)}\,\sqrt{1-\mu^2}}\times$$
$$\times \exp\left\{-\Sigma(E)r\mu - (\Sigma(E_0)-\alpha(E)\Sigma(E))r\sqrt{\frac{1-\mu^2}{1-\alpha^2(E)}}\right\},$$

where

$$r = |\mathbf{x}|,\qquad \mu = \frac{\mathbf{x}}{|\mathbf{x}|},\qquad \alpha(E) = \frac{A+1}{2}\sqrt{\frac{E}{E_0}} - \frac{A-1}{2}\sqrt{\frac{E_0}{E}}.$$

This formula is also generalized to the case in which the medium consists of a mixture of several elements.

In carrying out numerical calculations it is often necessary to obtain quantities

$$I_Q = \int_Q f_1(\mathbf{x},\ ,E)\,\varphi(E,\mu)\,d\mu\,dE$$
$$= \text{const}\int_Q \frac{\varphi(E,\mu)\,d\mu\,dE}{\sqrt{1-\mu^2}\sqrt{1-\alpha^2(E)}}\times$$
$$\times \exp\left\{-\Sigma(E)r\mu - (\Sigma(E_0)-\alpha(E)\Sigma(E))r\sqrt{\frac{1-\mu^2}{1-\alpha^2(E)}}\right\}.$$

For fixed r, the function $f_1(r,\mu,E)$ is nonzero in the curvilinear triangle shown in Fig. 1, and has singularities on two of its legs. These singularities are such that the integral (4) over the-

regions Q that touch the legs cannot be represented in the form of repeated one-dimensional integrals. Therefore, for the calculation of the integrals (4) over these regions, asymptotic formulas are required. By Laplace' s method, for the small regions shown in the figure, one can obtain the following asymptotic formulas:

$$I_{\mathrm{I}} \sim \mathrm{const}\,\frac{\varphi\left(E_0\left(\frac{A-1}{A+1}\right)^2,-1\right)}{\left[\Sigma(E_0)+\Sigma\left(E_0\left(\frac{A-1}{A+1}\right)^2\right)\right]r}\exp[-\Sigma(E_0)r]\varepsilon+o(\varepsilon),$$

$$I_{\mathrm{II}} \sim \mathrm{const}\,\frac{\varphi\left(E_0\left(\frac{A-1}{A+1}\right)^2,+1\right)}{\left[\Sigma(E_0)+\Sigma\left(E_0\left(\frac{A-1}{A+1}\right)^2\right)\right]r}\exp\left[-\Sigma\left(E_0\left(\frac{A-1}{A+1}\right)^2\right)r\right]\varepsilon+o(\varepsilon),$$

$$I_{\mathrm{III}} \sim \mathrm{const}\cdot\varphi(E_0,+1)\exp[-\Sigma(E_0)r]\varepsilon+o(\varepsilon),$$

$$I_{\mathrm{IV}} \sim \mathrm{const}\cdot\sqrt{2\delta}\int_{\Delta E}\frac{dE\,\varphi(E,+1)}{\sqrt{1-\alpha^2(E)}}\exp[-\Sigma(E)r]+o(\sqrt{\delta}),$$

$$I_{\mathrm{V}} \sim o(\varepsilon).$$

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Note: Figure translations are in progress. See original paper for figures.