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DETERMINATION OF THE VELOCITY PROFILE IN THE VISCOUS SUBLAYER ON THE BASIS OF THE PRINCIPLE OF MAXIMUM STABILITY

(Presented by Academician V. V. Struminskii, 30 X 1970)

The principle of maximum stability of averaged turbulent flows, used for calculating the Kármán constant of turbulent flow in a plane channel ([1,2]), led to results satisfactorily agreeing with experiment. In these specific calculations, it was assumed that the direct interaction of turbulent pulsations with a small disturbance introduced into the flow may be neglected in comparison with the effect on the disturbance of the profile of the averaged velocity.

This assumption is essential, since it makes it possible to reduce the problem to a numerical one, using the methods of the traditional theory of hydrodynamic stability and without touching upon the complex questions connected with the construction of a model for the interaction of pulsations and disturbances.

It is known ([3]) that various factors, such as a magnetic field or suction, affect the stability of plane-parallel wall-bounded laminar flows mainly through deformation of the velocity profile. The direct influence of these factors on the behavior of small disturbances, associated with the appearance of additional terms in the Orr–Sommerfeld equation, is insignificant.

One may expect that in wall-bounded turbulent flows the stabilizing action of turbulent pulsations is also manifested mainly through deformation of the profile of the averaged velocity. This is also indicated by the results of calculations ([1,2]). There are, however, cases (a submerged jet, a wake behind a poorly streamlined body) when laminar and turbulent velocity profiles are close in form, and the stability of the averaged turbulent flow is ensured precisely by the direct interaction of turbulent pulsations and external disturbances.

It seems reasonable, before attempting to take this interaction into account, which may be associated with new assumptions, to undertake an additional

Fig. 1

Figure 1: Fig. 1

verification of the principle of maximum stability under conditions in which one can deliberately restrict oneself to accounting for the interaction only with the averaged velocity profile. Such conditions do exist. As the authors'investigations have shown ([4]), the problem of hydrodynamic stability has the properties of locality.

Along with the global stability of the entire flow, which is determined by the behavior of long-wave disturbances with wavelength greater than or of the order of the characteristic dimensions of the flow, one may study the local stability of separate sections of the flow to short-wave disturbances with wavelength smaller than or of the order of the size of the section under consideration.

Here it is important to note that, in the case of attenuation of small disturbances, the characteristics of local and global stability (the corresponding decrements of attenuation) are practically independent of one another. Short-wave disturbances possess the property of finiteness, i.e., their amplitude is practically different from zero only on a small section of the flow of the order of a wavelength. A small local deformation of the velocity profile may

can have a substantial effect only on the behavior of short-wave disturbances localized in the zone of this deformation.

Most important is that, for short-wave disturbances whose dimensions are considerably smaller than the scales of the turbulent fluctuations responsible for momentum transfer, their behavior will be determined entirely by interaction with the mean flow and can obviously be described within the framework of the usual theory of hydrodynamic stability.

If one considers the entire spectrum of small disturbances, then, generally speaking, short-wave disturbances will be localized in all portions of the flow; but for convex profiles, characteristic of pressure flows in channels, the most dangerous short-wave disturbances (i.e., those that decay more slowly than the others) are located near the maximum of the mean velocity and at the channel walls ([4]). In ([5]) the near-axis region of the flow, where the velocity reaches a maximum, was considered, and it was shown that over a wide range of Reynolds numbers the distribution of mean velocity most stable with respect to short-wave disturbances is a semicubical parabola, which corresponds to the experimental Darcy law for a turbulent core.

Fig. 1

Here the local stability of the near-wall region will be considered, i.e., of the so-called viscous sublayer and buffer region. Outside this zone, a logarithmic law of velocity distribution is assumed. The class of profiles tested for stability will be chosen on the basis of the dynamical equation

$$\nu\, du/dy + l^2 |du/dy|\, du/dy = v_*^2, \tag{1}$$

where v_* is the dynamic velocity, u is the averaged flow velocity, ν is the coefficient of kinematic viscosity, y is the distance from the channel wall, and l is the mixing length.

Below, dimensionless quantities will be used:

$$u_+ = u/v_{*0}; \qquad y_+ = yv_{*0}/\nu; \qquad L = lv_{*0}/\nu; \qquad \mathrm{Re}_+ = v_{*0}h/\nu;$$

$$c_+ = X + iY = c/v_{*0}; \qquad \alpha_+ = \alpha\nu/v_{*0}; \qquad \alpha_1 = \alpha h = \alpha_+ \mathrm{Re}_+,$$

where v_{*0} is the value of the dynamic velocity at the wall, α is the wave number of the disturbance, c is the complex propagation velocity of the disturbance, h is the channel half-width, and Re_+ is the flow Reynolds number.

For the mixing length the dependence

$$L^2 = \varkappa^2 y_+^2 [1 - \exp(-y_+/A)]^n \tag{2}$$

is adopted.

It is a generalization of Van Driest' s approximate formula ([6]) (where $n = 2$) and contains two parameters, A and n, which here are determined from the condition of maximum stability.

The constant $\varkappa$ characterizes the logarithmic region and is considered known ($\varkappa = 0.4$). In the calculations the zone of deformation of the velocity profile was within the interval $0 < y_+ < 100$.

Figure 1 presents the dependence $Y(\alpha_1)$ (curve 1) for the most dangerous near-wall mode of the disturbance spectrum in the case of flow in a plane channel at $n = 1$, $A = 100$, $\mathrm{Re}_+ = 500$. The maximum of $Y(\alpha_1)$, located at $\alpha_1 \sim 10^2$, corresponds to disturbances whose wavelength is much

smaller h and which are responsible for the local stability of the near-wall zone. Curve 2 in Fig. 1 shows the position of the critical point y_c, i.e., the place where the phase velocity of the disturbance coincides with the local flow velocity and where the short-wave disturbance is localized (y_c is the root of the equation $u_+(y_+) = X$), as a function of the wave number. The most dangerous short-wave disturbances in this case have their critical point in the region $y_+ < 10$.

The position of the curve $Y(\alpha_1)$ in the region of the short-wave maximum determines the degree of stability of the near-wall region. The lower the curve lies, the more rapidly the short-wave disturbances decay on it. Figure 2 presents the

Fig. 2 and Fig. 3

Figure 2: Fig. 2 and Fig. 3

dependence $\Pi = \max_\alpha Y$ on the parameter $B = A^n/\lambda^2$ for $n = 1$ (curve 1) and $n = 2$ (curve 2). The parameter B determines the mixing length for $y_+ \ll A$:

Fig. 2 **Fig. 3**

$$L^2 \simeq y_+^{2+n}/B. \tag{3}$$

In both cases the most stable velocity distribution corresponds to $B \approx 640$, but curve 1 lies considerably lower. This indicates that maximum stability corresponds to $n = 1$. Calculations were carried out in which n was assigned a noninteger value in the interval $0 < n < 2$; in this case as well, maximum stability was observed for n close to 1.

We note that the case $n = 1$ corresponds to a cubic dependence of the turbulent viscosity $l^2|du/dy|$ on the distance to the walls at the beginning of the buffer zone, whereas the Van Driest approximation ($n = 2$) gives the fourth power. Recently, the question of what this power should be has been widely discussed ([7,8]). The present work shows that the cubic dependence corresponds to the most stable velocity distribution.

Throughout the investigated range of Reynolds numbers $500 \leq \mathrm{Re}_+ \leq 1200$, the parameters $n = 1$, $A = 100$ corresponded to the maximally stable velocity profile. In this case the most dangerous disturbance had wavelength $\lambda = 2\pi/\alpha = 69$ and was localized in the neighborhood of $y_+ \approx 7$.

The longitudinal scale of the disturbance is substantially larger than the transverse one. The same elongated character of the vortices in the viscous sublayer was recorded in an experimental study of the near-wall zone ([9]); however, the longitudinal scale of the turbulent pulsations penetrating into the sublayer was considerably larger, $\lambda \sim 300 \div 400$.

In Fig. 3, curve 1 represents the most stable velocity distribution in the viscous sublayer and buffer zone. The dashed line 2 corresponds to the experimental profile averaged over a large body of data,

profile (7). The points mark data obtained directly at the Institute of Thermophysics, Siberian Branch of the USSR Academy of Sciences (10). Curve 3 corresponds to the linear velocity distribution $u_+ = y_+$.

Thus, the wall region of the universal velocity profile, calculated on the basis of the principle of maximum stability, is in good agreement with the experimental data.

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Note: Figure translations are in progress. See original paper for figures.