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Abstract

Full Text

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EFFECT OF NONUNIFORMITY OF THE ELASTIC FIELD OF SUPPORTS ON PARAMETRIC OSCILLATIONS OF A RIGID SHAFT

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To reduce vibration overloads in the bearing supports of rotating shafts, to decrease the amplitudes of oscillations at operating speeds and when passing through critical speeds, and to increase service life in modern high-speed machines ([1,2]), elastic supports are used, the load-bearing elements of which in a number of cases are made in the form of elastic rings (Fig. 1). Elastic rings ([3]) have a number of advantages over other types of elastic load-bearing elements: compactness of design, sufficient stiffness, durability, and the possibility of accurate calculation.

On the other hand, the stiffness field of elastic rings is nonuniform. Nonuniformity of the elastic field of a support may be the cause of parametric oscillations. To determine a way of eliminating such vibrations, let us consider small oscillations of an absolutely rigid rotor mounted on rolling bearings in one elastic and one second, rigid, hinged support. Suppose that the stiffness coefficient of the support is linear with respect to radius but nonlinear with respect to angle, and that k is the number of periods of variation of the stiffness coefficient when the rotor center turns through one revolution. Denoting by ω and ω_0, respectively, the angular velocities of rotation of the rotor and of its precession, one may approximately interpolate the law of variation of the stiffness of the elastic field of the support by the function

Fig. 1. Types of rings

$$c(t) = c[1 + n\cos k(\omega_0 t - \alpha_0)],$$

where $n = \Delta c/c$ is the relative variation of the stiffness coefficient of the support.

Since the forced oscillations of the rotor are, as a rule, caused by static and dynamic unbalances of the rotor, we set $\omega_0 = \omega$. In this case the differential equations of the forced oscillations of the rotor, taking into account the action of gyroscopic and damping forces, have the form

$$(B + ml_1^2)\ddot{z} - A\omega\dot{y} + \delta\dot{z} + zl^2c[1 + n\cos k(\omega t - \alpha_0)] = mll_1e\omega^2\sin\omega t,$$

$$(B + ml_1^2)\ddot{y} + A\omega\dot{z} + \delta\dot{y} + yl^2c[1 + n\cos k(\omega t - \alpha_0)] = mll_1e\omega^2\cos\omega t, \qquad (1)$$

differing from the equations considered in (2) by the presence of terms that take into account the nonuniformity of the elastic field. In these equations m is the mass of the rotor; e is the static unbalance; A and B are the central axial and equatorial moments of inertia of the rotor; l is the distance between supports; l_1 is the distance from the center of gravity of the rotor to the hinged support; δ is the co-

coefficient of linear viscous damping for displacements in any direction perpendicular to the rotor axis.

Introducing the notation

$$\gamma = \delta/(B + ml_1^2); \qquad a = A/(B + ml_1^2), \qquad \omega_* = \sqrt{cl^2/(B + ml_1^2)};$$

$$h = mll_1/(B + ml_1^2),$$

we write system (1) in the form

$$\ddot{z} - a\omega\dot{y} + \gamma\dot{z} + \omega_*^2[1 + n\cos k(\omega t - \alpha_0)]z = eh\omega^2\sin\omega t, \qquad (2)$$

$$\ddot{y} + a\omega\dot{z} + \gamma\dot{y} + \omega_*^2[1 + n\cos k(\omega t - \alpha_0)]y = eh\omega^2\cos\omega t.$$

The solution of this linear system of nonhomogeneous differential equations with periodically varying coefficients was carried out by the method of mathematical modeling on an analog computer. For this purpose system (2) was transformed to a new, dimensionless time $\tau = \omega_* t$ and to dimensionless displacements of the center of the elastic support $\bar{z} = z/l$ and $\bar{y} = y/l$:

$$\ddot{\bar{z}} - a\beta\dot{\bar{y}} + D\dot{\bar{z}} + [1 + n\cos k(\beta\tau - \alpha_0)]\bar{z} = h\beta^2\sin\beta\tau, \qquad (3)$$

Fig. 2. Amplitude-frequency characteristic in the plane of dimensionless speeds $\beta = \omega/\omega_*$ and amplitudes $\rho = \rho/e$. The hatched region is the region of parametric resonance

Figure 2: Fig. 2. Amplitude-frequency characteristic in the plane of dimensionless speeds $\beta = \omega/\omega_*$ and amplitudes $\rho = \rho/e$. The hatched region is the region of parametric resonance

$$\ddot{\bar{y}} + a\beta\dot{\bar{z}} + D\dot{\bar{y}} + [1 + n\cos k(\beta\tau - \alpha_0)]\bar{y} = h\beta^2\cos\beta\tau,$$

where $D = \gamma/\omega_*$; $\beta = \omega/\omega_*$.

It should be noted that the dimensionless coefficients of system (3), a, D, h, n, vary within very narrow limits for a broad class of rotors encountered in practice, regardless of their rotational speeds and geometric characteristics.

Thus, solving system (3) makes it possible to represent the behavior of rotors in a very general formulation. The solution was carried out for $k = 3$, which corresponds to the rings shown in Fig. 1 and is the most complicated case.

Fig. 2. Amplitude-frequency characteristic in the plane of dimensionless speeds $\beta = \omega/\omega_*$ and amplitudes $\rho = \rho/e$. The hatched region is the region of parametric resonance

To find the zones of parametric resonances, on the time scale $M_\tau = \tau_m/\tau = 2$ (where τ_m is machine time), at the greatest inhomogeneity of the elastic field of the support ($n = 1$) and minimal viscous friction ($D = 0.025$), the range of dimensionless rotor speeds from zero to $\beta = 8$ was traversed. Only one zone of unstable oscillations was found in the range of dimensionless speeds from zero to $\beta = 1$ (Fig. 2).

Attempts to find secondary zones of parametric resonance also on other time scales $M_\tau = 1.5;\ 1.0;\ 0.5;\ 0.25$ were unsuccessful. This is explained by the fact that the secondary regions narrow substantially under the influence of damping, and, for sufficiently small excitation coefficients, the presence of damping makes the occurrence of parametric resonance impossible [4]. The second, and still more the third, regions of instability can be detected on an analog computer only when operating in this zone in a steady-state regime, i.e., with zero acceleration, which -

···to carry out is practically very difficult. Under real conditions these zones do not present a danger (^{4}).

Oscillographic recording of the trajectories of the center of the transverse section of the shaft corresponding to the center of the elastic support, at various points of the amplitude-frequency characteristic (Fig. 2), makes it possible to assert that the nature of the rotor motion in the region of parametric instability does not even remotely resemble the closed trajectories of stable rotational regimes and gives a picture of an unstable process (Fig. 2, A). In the off-resonance region (at $\beta = 2$) the trajectory of the same point is a circle (Fig. 2, B) and, despite the great inhomogeneity of the elastic field ($n = 1$), is identical to the trajectory for a homogeneous elastic field of the support.

The regions of instability of the system, shown in the plane of the dimensionless parameters n and β (Fig. 3), make it possible to draw the following conclusions.

An increase in the gyroscopic terms of the system (a) has practically no effect on the width of the instability zone

Fig. 3 Fig. 4

Fig. 3. Regions of instability in the β, n plane for $D = 0.25$ (1); 0.2 (2); 0.15 (3); 0.1 (4); 0.05 (5) and 0.025 (6, 7, and 8); $a = 0.1$ (I); 0.3 (II) and 0.5 (III)

Fig. 4. Regions of stability in the ε, D plane for $n = 0.5$ (1); 0.4 (2) and 0.3 (3); $h = 1.0,\ \rho_{\max} = 10$

and the lower limit of the inhomogeneity of the elastic field of the support at which the appearance of parametric resonance is possible (Fig. 2).

With an increase in the relative damping coefficient D, the regions of instability narrow. The presence of damping, as it were, “cuts off” those parts of the instability region that adjoin the abscissa axis.

From the expressions

$$D = \delta/l\sqrt{B + ml_1^2}\sqrt{c}, \qquad \Delta\beta = \Delta\omega\sqrt{B + ml_1^2}/l\sqrt{c}$$

it follows that a decrease in the stiffness coefficient of the elastic field of the support, with the other parameters of the system unchanged, contributes to lowering the level of parametric vibrations of the shaft and makes it possible, even with natural damping and without designing special dampers, to successfully pass through the zone of possible parametric vibrations of the system.

Since, for a rotor rotating in elastic and journal bearings, the zone of operating speeds lies above the first critical rotational speed, the main region of parametric vibrations will be traversed during run-up and run-down.

A study of the parametric oscillations of the rotor at various dimensionless angular accelerations ($\bar{\varepsilon} = \varepsilon/\omega_*^2$) shows that these oscillations, having arisen in the zone of parametric resonance, disappear after passing through the instability zone. A certain persistence of the parametric oscillations was also observed, but by the time the first critical speed was passed these oscillations always disappeared. Studies of oscillations during rotor run-up also show that there exists an upper limit of the angular acceleration of the rotor at which parametric oscillations do not have time to develop.

In Fig. 4 the "stability" regions of the system are plotted in the plane of the dimensionless parameters $\bar{\varepsilon}, D$. Here "stability" is understood conventionally as such a regime of passage through the parametric-resonance zone in which the amplitude of the parametric oscillations does not exceed 10 unbalances. It follows from Fig. 4 that the stability zone expands as the nonuniformity of the elastic field decreases and, practically at the value $n = 0.3$, occupies almost the entire plane of the dimensionless parameters $\bar{\varepsilon}, D$.

On the other hand, by increasing the rate of rotor acceleration for a given nonuniformity, one can always successfully pass through the zone of parametric oscillations.

Thus:

1. Owing to the nonuniformity of the elastic field of the support, parametric oscillations may arise in rotors, accompanied by increased vibrations in the pre-resonance region.
2. The nonuniformity of the elastic field of the support in the post-resonance region, i.e., in the range of operating rotor speeds, affects neither the character of the rotor precession nor the amplitude of the oscillations.
3. The gyroscopic terms of the system do not affect the width of the parametric-resonance zone.
4. Linear viscous damping and a decrease in the stiffness of the supports narrow the zone of parametric oscillations and reduce their level.

For combating parametric oscillations two approaches may be used: reducing the stiffness coefficient of the elastic support and increasing the rate of acceleration and braking of the rotor.

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CITED LITERATURE

1. A. S. Kel' zon, DAN, **110**, No. 1 (1956).
2. A. S. Kel' zon, V. I. Pryadilov, Izv. AN SSSR, Mechanics, No. 6 (1965).
3. A. S. Kel' zon, V. I. Pryadilov, Author' s Certificate No. 182968, Bulletin of Inventions, Industrial Designs, Trademarks, No. 12 (1966).
4. V. V. Bolotin, *Dynamic Stability of Elastic Systems*, Moscow, 1956.

Note: Figure translations are in progress. See original paper for figures.