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Abstract

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ON THE COMPTON MECHANISM OF HEATING AN ELECTRON GAS BY MEANS OF LASERS

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In connection with the continuous increase in the intensities of electromagnetic fields obtained by means of lasers, it is of interest to study the physical effects responsible for absorption of the energy of laser radiation. This question is also important from an applied point of view, for example in connection with the possibility of heating a plasma to thermonuclear temperatures in the focus of a laser.

The mechanisms playing the dominant role in the absorption of laser radiation in various ranges of its intensity were considered in the review ([1]). There, in particular, the case of such high radiation intensities was considered qualitatively, when the energy of oscillations of an electron in the external laser field becomes comparable with its rest energy mc^2 (in the optical range this corresponds to a radiation energy flux $\sim 10^{18}$ W/cm^2). It was shown that in this case an electron plasma absorbs energy mainly at the expense of relativistic effects, above all at the expense of the multiphoton Compton effect in an external field, consisting in the absorption from the laser field of several photons and the emission of one hard photon. Estimates were given from which it follows that the recoil energy of the electron in this case may considerably exceed the recoil energy in the ordinary Compton effect and that the absorption coefficient of intense laser radiation by a plasma may be significant.

In the present work the rate of absorption of energy by an electron gas and the absorption coefficient of intense laser radiation due to the multiphoton Compton effect are calculated.

The cross sections of the Compton effect in an external field were obtained in works ([2–6]). According to the results of these works, the magnitude of the external field is characterized by the relativistically invariant parameter

$$x = e\sqrt{a^2}/m. \tag{1}$$

Here a_μ is the amplitude of the 4-potential of this field ($a^2 = \mathbf{a}^2 - a_0^2$); e, m are the charge of the electron and its mass (units are used in which $c = 1$, $\hbar = 1$, $e^2 = 1/137$).

An electron in the field of a plane monochromatic electromagnetic wave is described by a Volkov wave function ([7,8]), while the role of the time-averaged 4-momentum is played by the so-called 4-quasimomentum q_μ, whose square is

$$q^2 = -m_*^2 = -m^2(1 + x^2).$$

Here m_* is the effective mass of the electron in the external field (the addition to the mass is due to the energy of oscillations of the electron in the external field).

To simplify the calculations it is sufficient to restrict ourselves to the case of a circularly polarized plane monochromatic electromagnetic wave with 4-potential:

$$A_\mu = a_{1\mu} \cos kx + a_{2\mu} \sin kx, \tag{2}$$

where k_μ is the wave 4-vector, $kx = \mathbf{kx} - \omega x_0$, $ka_1 = ka_2 = a_1 a_2 = 0$, $k^2 = 0$, $a_1^2 = a_2^2 \equiv a^2$.

For it, in (3) the following expression was obtained for the probability of photon emission by an electron (per unit volume per unit time)

$$\begin{gathered} W(\chi, x) = \sum_{s=1}^{\infty} W_s = \frac{e^2 m^2 n}{4q_0} \sum_{s=1}^{\infty} \int_0^{u_s} \frac{du}{(1+u)^2} \left\{ -4J_s^2(z) + \right. \\ \left. + x^2 \left(2 + \frac{u^2}{1+u} \right) \left(J_{s-1}^2 + J_{s+1}^2 - 2J_s^2 \right) \right\}, \end{gathered} \tag{3}$$

$$u = \frac{kk'}{kq'}, \qquad u_s = -\frac{2s(kq)}{m_*^2} = \frac{2s\chi}{x(1+x^2)},$$

$$z = 2s \frac{x}{\sqrt{1+x^2}} \sqrt{\frac{u}{u_s}\left(1 - \frac{u}{u_s}\right)};$$

k', q' are, respectively, the 4-momentum of the emitted photon and the 4-quasimomentum of the electron in the final state, $J_s(z)$ is the Bessel function, and n is the number of electrons per unit volume.

The probability (3) depends on two invariants x and $\chi = -x(kp)/m^2$, where p_μ is the constant 4-vector determining the Volkov state of the electron, with

$$q_\mu = p_\mu - \frac{e^2 a^2}{2(kp)} k_\mu, \qquad p^2 = m^2.$$

Each term W_s of the sum (3) is the probability of a process occurring with absorption of s photons from the laser field. In this case the obvious conservation laws are satisfied:

$$sk + q = k' + q'. \tag{4}$$

In [3] the 4-momentum emitted per unit volume per unit time was also calculated. After elementary transformations using the conservation laws (4), its expression can be written in the form

$$\begin{aligned} I_\mu^{\text{em}} = \frac{e^2 m^2 n}{4q_0} \sum_{s=1}^{\infty} \int_0^{u_s} \left\{ p_\mu + k_\mu \left(\frac{s}{u} - \frac{1}{2}\frac{x^3}{\chi} - \frac{x}{\chi} \right) \right\} \times \\ \times \frac{u}{(1+u)^3} \left\{ -4J_s^2(z) + x^2 \left(2 + \frac{u^2}{1+u} \right) (J_{s-1}^2 + J_{s+1}^2 - 2J_s^2) \right\} du. \end{aligned} \tag{5}$$

Subtracting I_μ^{em} from the 4-momentum I_μ^{abs} absorbed per unit volume per unit time and equal to $k_\mu \sum_{s=1}^{\infty} sW_s$, we obtain the expression for the 4-momentum acquired by a unit volume of the electron gas per unit time:

$$\begin{aligned} I_\mu = I_\mu^{\text{abs}} - I_\mu^{\text{em}} = \frac{e^2 m^2 n}{4q_0} \sum_{s=1}^{\infty} \int_0^{u_s} du \, \frac{u}{(1+u)^3} \times \\ \times \left\{ \left[k_\mu \left(s + \frac{1}{2}\frac{x^3}{\chi} + \frac{x}{\chi} \right) - p_\mu \right] \right\} \left\{ -4J_s^2(z) + x^2 \left(2 + \frac{u^2}{1+u} \right) \times \right. \\ \left. \times (J_{s-1}^2 + J_{s+1}^2 - 2J_s^2) \right\}. \end{aligned} \tag{6}$$

This expression does not admit analytical calculation.

If we restrict ourselves to the practically interesting case of an initially not very hot electron gas (with initial temperature $T \ll 10^8$ degrees), then the electrons in the initial state are nonrelativistic (their kinetic energy $\sim kT \ll 10^4$ eV $\ll mc^2 = 5 \cdot 10^5$ eV). Then

$$kp = \omega\sqrt{p_0^2 - m^2}\cos\theta - \omega p_0 \simeq -\omega p_0 \simeq -\omega m,$$

$$\chi = -x\,\frac{kp}{m^2} \simeq x\,\frac{\omega}{m} \ll x. \tag{7}$$

(In the optical range, $\omega/m \sim 10^{-6}$.)

$$u_s = 2s\,\frac{x}{x(1+x^2)} \simeq 2s\left(\frac{\omega}{m}\right)\frac{1}{1+x^2}. \tag{8}$$

In this case, from expression (6) for the recoil energy I_0, the dependence on the initial plasma temperature drops out in the first approximation, i.e., on the initial kinetic energy of the electrons (in this approximation one may assume that the initial electrons were at rest: $p_0 = m$). Then the expression for the recoil energy takes the form:

$$I_0 = \frac{e^2 m^2 n}{4}\left(\frac{\omega}{m}\right)\frac{1}{1+\frac{1}{2}x^2}\sum_{s=1}^{\infty}\left[s + \frac{1}{2}x^2\left(\frac{m}{\omega}\right)\right]\times$$

$$\times\int_0^{u_s} du\,\frac{u}{(1+u)^3}\left\{-4J_s^2(z) + x^2\left(2+\frac{u^2}{1+u}\right)\left(J_{s-1}^2 + J_{s+1}^2 - 2J_s^2\right)\right\}. \tag{9}$$

In papers (2-5) it was pointed out that for $x \gg 1$, in each elementary act on the order of x^3 photons are absorbed from the laser field; in other words, in the series (9) one must take into account very many ($\sim x^3$) terms. At the same time, the case of not small x is of physical interest, since the energy $\hbar\omega$ of a laser quantum is fairly small ($\sim$ 1-2 eV), and therefore the plasma can be strongly heated only at such radiation intensities for which the processes taking place are essentially multiphoton.

Table 1

x	$\dot{\varepsilon}$, eV/sec	α, cm^{-1}
0.1	$4.06 \cdot 10^{10}$	$2.75 \cdot 10^{-5}$
0.5	$5.39 \cdot 10^{11}$	$1.45 \cdot 10^{-5}$
1	$1.21 \cdot 10^{12}$	$0.82 \cdot 10^{-5}$
2	$2.23 \cdot 10^{13}$	$3.78 \cdot 10^{-5}$
3	$6.38 \cdot 10^{13}$	$4.8 \cdot 10^{-5}$
4	$1.38 \cdot 10^{14}$	$5.83 \cdot 10^{-5}$
5	$2.62 \cdot 10^{14}$	$7.1 \cdot 10^{-5}$
6	$4.54 \cdot 10^{14}$	$8.5 \cdot 10^{-5}$
7	$7.25 \cdot 10^{14}$	$1.0 \cdot 10^{-4}$
8	$1.09 \cdot 10^{15}$	$1.15 \cdot 10^{-4}$
9	$1.54 \cdot 10^{15}$	$1.29 \cdot 10^{-4}$
10	$2.14 \cdot 10^{15}$	$1.45 \cdot 10^{-4}$

Expression (9) was calculated on an electronic computer for values of the parameter x in the range 0.1–10; for the Bessel functions, asymptotic expressions (for $s \gg 1$) in terms of Airy functions were used (9, 10). The calculation was performed for the case of a laser on neodymium glass ($\hbar\omega = 1.17$ eV). The results of the calculation are summarized in Table 1, which gives the obtained values of the rate of energy acquisition $\dot{\varepsilon}$ (per electron), as well as the absorption coefficient $\alpha = I_0/S$ of the laser-radiation energy in an electron gas with density $n = 10^{20}$ cm^{-3} (S is the radiation intensity).

From the data presented it is evident that, in strong electromagnetic fields, an electron gas can be strongly heated due to the multiphoton Compton effect. For example, at the value of the parameter $x = 10$ (which corresponds to a laser-radiation energy flux $\sim 2 \cdot 10^{20}$ W/cm^2), during a picosecond (10^{-12} sec) laser pulse the electron gas can be heated to 10^7 degrees, i.e., to thermonuclear temperatures.

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Note: Figure translations are in progress. See original paper for figures.