ON FINDING POSITIONS OF THE REDUCTION LINK THAT ARE EXTREMAL FOR THE CHARACTERISTIC CRITERION OF THE PERIODIC REGIME OF MOTION OF A MACHINE UNIT

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Abstract

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ON FINDING POSITIONS OF THE REDUCTION LINK THAT ARE EXTREMAL FOR THE CHARACTERISTIC CRITERION OF THE PERIODIC REGIME OF MOTION OF A MACHINE UNIT

1. In the work ([1]) the importance was indicated of the problem of finding those positions of the reduction link of a machine unit in which the influence of the inertia forces of the initial motion on the links of the machine proves most substantial in comparison with the inertia forces of permanent motion, in the sense of N. E. Zhukovskii ([2]). In these positions the characteristic criterion $\chi_\xi(\varphi)$ of the ξ-periodic limiting regime $T = T_\xi(\varphi)$ of motion of a machine unit ([3]), expressing the law of distribution of inertial forces between the indicated motions, takes extremal values.

In the present article we wish to dwell on possible methods of seeking the indicated positions of the reduction link, since they are computational in estimating those additional dynamic loads on the links of the machine which are caused by the inertial forces of the initial motion.

Following the terminology and notation adopted in article ([3]), we write the equation of motion of a machine unit in the form

$$dT/d\varphi = M(\varphi, T), \tag{1}$$

assuming that:

1°. The reduced moment $M(\varphi, T)$ of all acting forces is a function defined and continuous in the strip

$$0 \le T \le \hat{T}, \qquad -\infty < \varphi < +\infty, \tag{2}$$

where $\hat{T}$ is the maximum possible value of the kinetic energy of the machine unit that the acting forces can impart to it.

2°. $M(\varphi, 0) > 0,\ M(\varphi, \hat{T}) \le 0.$

3°. The slope of the reduced moment of all acting forces is continuous and negative in the strip (2), $M'_T(\varphi, T) < 0$.

4°. The reduced moment $M(\varphi, T)$ has period ξ with respect to the angle of rotation φ: $M(\varphi + \xi, T) = M(\varphi, T)$.

We shall regard the reduced moment of inertia of the masses of all links as a function of the angle of rotation of the reduction link, $I = I(\varphi)$.

Under the conditions considered, the slope of the reduced moment of all acting forces in the strip (2) is bounded below and above by certain negative constants

$$-\lambda_2 \leq M'_T(\varphi, T) \leq -\lambda_1 \qquad (0 < \lambda_1 \leq \lambda_2). \tag{3}$$

Introduce the notation

$$\tau_* = \inf_{0 \leq \varphi < \xi} \tau(\varphi), \qquad \tau^* = \sup_{0 \leq \varphi < \xi} \tau(\varphi), \tag{4}$$

where $T = \tau(\varphi)$ is the inertial curve of motion of the machine unit ([4]).

2. Under the assumptions made there exists, and moreover uniquely, a ξ-periodic limiting regime $T = T_\xi(\varphi)$ of motion of the machine aggre-

gate, contained entirely within the stability band,

$$\tau_* \leqslant T_\xi(\varphi) \leqslant \tau^*, \qquad -\infty < \varphi < +\infty. \tag{5}$$

Let us consider the corresponding logarithm

$$f(\varphi) = \ln T_\xi(\varphi)/I(\varphi), \qquad -\infty < \varphi < +\infty, \tag{6}$$

of the normalized kinetic energy of the machine aggregate.

Taking (5) into account, the characteristic criterion $\chi_\xi(\varphi)$ of the periodic limiting regime $T = T_\xi(\varphi)$ of motion of the machine aggregate is represented in the form

$$\chi_\xi(\varphi) = \left[\ln T_\xi(\varphi)/I(\varphi)\right]'_\varphi = f'(\varphi). \tag{7}$$

Therefore

$$d\chi_\xi(\varphi)/d\varphi = \left[\ln T_\xi(\varphi)/I(\varphi)\right]'' = f''(\varphi). \tag{8}$$

From the relations obtained it follows directly that the stationary points of the characteristic criterion $\chi_\xi(\varphi)$ coincide with the critical points of second order of the logarithm of the normalized kinetic energy of the machine aggregate which it develops in the periodic regime of motion.

In intervals of variation of the angle of rotation φ of the reduction link in which curve (6) is directed with its concavity upward (downward), the characteristic criterion $\chi_\xi(\varphi)$ increases (decreases).

At those critical points of second order of curve (6) which are points of inflection, the characteristic criterion $\chi_\xi(\varphi)$ has local extrema. In this case, at points of inflection at which the logarithm of the normalized kinetic energy changes the direction of concavity upward (downward) to the direction of concavity downward (upward), the characteristic criterion $\chi_\xi(\varphi)$ has a local maximum (minimum).

Consequently, the positions of the reduction link in which the inertia forces caused by the nonuniformity of the machine motion will exert the most substantial influence on its links relative to the inertia forces of permanent motion can be found from the points of inflection of the logarithm (6) of the normalized kinetic energy.

At those points of curve (8) where it crosses the axis $O\varphi$ from below upward (from above downward), the characteristic criterion $\chi_\xi(\varphi)$ of the periodic limiting regime $T = T_\xi(\varphi)$ has local minima (maxima).

3. The conclusions obtained in integrable cases are checked directly.

As an example one may consider a vertical cylindrical rotor, whose motion is described by the equation

$$I\, d\omega/dt = a + b\sin\varphi - k\omega^2, \tag{9}$$

where I is the moment of inertia of the rotor with respect to the axis of rotation; $M_d = a + b\sin\varphi$ $(a > b > 0)$, $M_c = k\omega^2$, respectively, are the moments of the driving forces and of the resistance forces applied to the rotor.

If we introduce the kinetic energy $T = I\omega^2/2$ of the rotor, then the equation of its motion can be written in the form

$$\frac{dT}{d\varphi} = a + b\sin\varphi - \frac{2k}{I}T. \tag{10}$$

We shall represent the periodic limiting regime of motion of the rotor in the form

$$T_{2\pi}(\varphi) = \frac{Ia}{2k} + \frac{Ib}{\sqrt{4k^2 + I^2}}\cos(\varphi - \varphi_0), \tag{11}$$

where $\varphi_0 = \pi - \operatorname{arctg} 2k/I$.

Restricting ourselves to the interval $0 \le \varphi \le 2\pi$, it is not difficult to verify that the points

$$\varphi_1 = \arccos \frac{2bk}{a\sqrt{4k^2 + I^2}} - \operatorname{arctg} \frac{2k}{I},$$

$$\varphi_2 = 2\pi - \operatorname{arctg} \frac{2k}{I} - \arccos \frac{2bk}{a\sqrt{4k^2 + I^2}} \tag{12}$$

are points of inflection of the logarithm

$$f(\varphi) = \ln \frac{T_{2\pi}(\varphi)}{I} = \ln \left[\frac{a}{2k} + \frac{b}{\sqrt{4k^2 + I^2}} \cos(\varphi - \varphi_0) \right] \tag{13}$$

of the normalized kinetic energy of the rotor.

At the first of them the characteristic criterion $\chi_{2\pi}(\varphi)$ of the periodic regime of rotor motion has a maximum, and at the second—a minimum, equal respectively to

$$\chi_{2\pi}(\varphi_1) = 2bk / \sqrt{a^2(4k^2 + I^2) - 4b^2k^2},$$

$$\chi_{2\pi}(\varphi_2) = -2bk / \sqrt{a^2(4k^2 + I^2) - 4b^2k^2}. \tag{14}$$

For the numerical data $I = 1$ kg $\cdot$ m^2, $a = 90$ Nm, $b = 40$ Nm, $k = 0.1$ Nm $\cdot$ sec^2, these results are illustrated in Fig. 1. For convenience, the graph of the logarithm of the normalized kinetic energy is shifted parallel to itself downward by 6 units from its actual position.

4. In the general case the periodic limiting regime $T = T_\xi(\varphi)$ of motion of the machine aggregate is not computed in finite form by quadratures.

In this connection, starting from an arbitrarily chosen continuous ξ-periodic function $T_1(\varphi)$ satisfying the inequality

$$\tau_* \le T_1(\varphi) \le \tau^*, \qquad -\infty < \varphi < +\infty, \tag{15}$$

construct the functional sequence $T_k(\varphi)$ according to the recurrence law

$$T_{k+1}(\varphi) = \frac{-e^{-\lambda_2 \varphi}}{e^{\lambda_2 \xi} - 1} \int_\varphi^{\varphi+\xi} e^{\lambda_2 t} \{ M[t, T_k(t)] + \lambda_2 T_k(t) \} \, dt, \tag{16}$$

$$k = 1, 2, 3, \dots, \qquad -\infty < \varphi < +\infty.$$

The latter converges uniformly [4] on the entire number line to the periodic limiting regime $T = T_\xi(\varphi)$ of motion of the machine aggregate,

$$T_k(\varphi) \to T_\xi(\varphi), \qquad -\infty < \varphi < +\infty, \tag{17}$$

as $k \to \infty$.

By analogy with expression (6), for the logarithm of the normalized kinetic energy we compose the sequence of functions

$$f_k(\varphi) = \ln T_k(\varphi)/I(\varphi), \qquad -\infty < \varphi < +\infty, \quad k = 1, 2, 3, \ldots . \tag{18}$$

From (6) and (18) we obtain

$$|f_k(\varphi) - f(\varphi)| = |\ln T_k(\varphi)/T_\xi(\varphi)|. \tag{19}$$

It is not difficult to verify that, whatever positive number ε is taken, the inequality

$$|\ln T_k(\varphi)/T_\xi(\varphi)| < \varepsilon, \qquad -\infty < \varphi < +\infty, \tag{20}$$

will hold for all sufficiently large indices k.

Therefore the sequence $f_k(\varphi)$ converges uniformly on the entire number line to the logarithm of the normalized kinetic energy of the machine-

of the aggregate:

$$f_k(\varphi) \rightrightarrows f(\varphi), \qquad -\infty < \varphi < +\infty, \tag{21}$$

as $k \to \infty$.

Within the limits of the required degree of accuracy, for sufficiently large k we may set

$$f(\varphi) = f_k(\varphi), \qquad -\infty < \varphi < +\infty. \tag{22}$$

Then the search for the positions of the reduction member that are extremal for the characteristic criterion $\chi_\xi(\varphi)$ of the periodic limiting regime of motion of the machine aggregate reduces to finding the inflection points of curve (18).

[Figure labels: χ; $\mu_\chi = \frac{0.004}{\text{mm}}$; $f(\varphi) = \ln \frac{T_{\xi z t}(\varphi)}{I}$; $\chi_{2\pi}(\varphi)$; $\frac{d\chi_{2\pi}(\varphi)}{d\varphi} = f''(\varphi)$; φ; 2π; $\mu_\varphi = \frac{\pi}{40}\,\frac{\text{rad}}{\text{mm}}$.]

Fig. 1

5. If dependence (6) is presented graphically, then finding the positions of the reduction member in which the characteristic criterion $\chi_\xi(\varphi)$ assumes extremal values can be carried out after double graphical differentiation of (6), at the points of intersection of the curve

$$f''(\varphi) = \left[\ln T_\xi(\varphi)/I(\varphi)\right]''_\varphi$$

with the axis $O\varphi$.

However, this method, like the determination of the indicated positions directly from the graph of the characteristic criterion $\chi_\xi(\varphi)$, is usually associated with large errors. This is especially true for machine aggregates with small coefficients of nonuniformity of motion.

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[1] I. I. Artobolevskii, *Izv. AN SSSR, OTN*, No. 12 (1952). [2] N. E. Zhukovskii, *Complete Works*, 1, 1937. [3] I. I. Artobolevskii, V. S. Loshchinin, *DAN*, 186, No. 3 (1969). [4] V. S. Loshchinin, *Tr. Inst. Mashinovedeniya, Seminar po teorii mashin i mekhanizmov*, 23, p. 91, Publishing House of the Academy of Sciences of the USSR, 1961. [5] I. I. Artobolevskii, V. S. Loshchinin, *DAN*, 186, No. 2 (1969).

Note: Figure translations are in progress. See original paper for figures.