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Abstract

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MATHEMATICS

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ON THE EXISTENCE OF FUNDAMENTAL DOMAINS OF A FINITELY GENERATED KLEINIAN GROUP

(Presented by Academician M. A. Lavrent'ev, June 2, 1969)

§ 1. B. Maskit in the paper ⁽¹⁾ formulated the problem of the existence of a fundamental domain of a Kleinian group with a finite number of generators. Below a solution of this problem is given.

We shall give the definitions and results known to us that are needed. By a fundamental domain in a locally compact metric space Y with respect to a discrete group of homeomorphisms G we shall mean an open set $F \subset Y$ satisfying the following two conditions: 1) for any $g_1 \neq g_2$ the sets $g_1\bar{F}$ and $g_2\bar{F}$, where \bar{F} is the closure of the set F , have no common elements; 2) the union of the sets gF , where g runs through the group G , coincides with the whole space Y .

A group of Möbius transformations is called discontinuous at a point z if z has a neighborhood U such that the set $g(U) \cap U = \emptyset$ for any $g \in G$. If a group of Möbius transformations is discontinuous at some point $z \in Y$, then it is called a Kleinian group. The set of points Y at which G is discontinuous forms the regular set $R(G)$. The set of points $z \in Y$ for which there exist a point $z_0 \in Y$ and a sequence of distinct elements $\{g_n\}$ from G having the property that $g_n(z) \rightarrow z_0$ as $n \rightarrow \infty$ is called the limit set $L(G)$.

It is known (see ⁽³⁾) that $R(G)$ is an open set, $L(G)$ is a closed set, and $R(G) \cap L(G) = \emptyset$. These sets are invariant with respect to G .

Fundamental domains of Fuchsian groups of the first and second kinds, which are special varieties of Kleinian groups, are described, for example, in ⁽²⁾.

§ 2. Our aim is to prove the following theorem.

Theorem. *A finitely generated Kleinian group has fundamental domains.*

Proof. Consider the quotient space $S = R(G)/G$. It has a natural complex structure such that the projection mapping $\pi : R(G) \rightarrow S$ is holomorphic. Thus, the components S_i of the quotient space S are Riemann surfaces. Let

$R_i = \pi^{-1}S_i$. Generally speaking, R_i are not connected. We shall denote the components of R_i by R_{ij} .

Thus, for any Kleinian group G we have associated decompositions

$$R = \bigcup_i R_i = \bigcup_{i,j} R_{ij}, \quad S = \bigcup_i S_i.$$

Each R_i is invariant with respect to transformations of the whole group G . The boundaries of R_i lie in $L(G)$. The components R_{ij} are branched covering surfaces over S_i , whose branch points are elliptic fixed points (see (3)).

Each connected component R_{ij} has at least three boundary points. In the case when the number of boundary points is less than three, we have a Fuchsian group, for which a fundamental domain exists. The R_{ij} have universal covering surfaces V_{ij} , which conform-

are mapped onto the disk $|w| < 1$. The Poincaré metric ρ , defined in the disk $|w| < 1$, is transferred to the universal covering surface V_{ij} and, in particular, to R_{ij} (see (4)). The metric ρ , defined on each R_{ij} , gives a metric on all of R . The projection π induces the corresponding metric $\bar{\rho}$ on S . This metric $\bar{\rho}$ has the property of intrinsicness, i.e., for any two points $y_1, y_2 \in Y$ there exists a third point $y_0 \in Y$ such that

$$\bar{\rho}(y_1, y_0) = \bar{\rho}(y_0, y_2) = \frac{1}{2}\bar{\rho}(y_1, y_2).$$

The locally compact metric space Y with such a metric $\bar{\rho}$ has fundamental domains (see (5)). The theorem is proved. This theorem gives a solution to the problem of B. Maskit stated above.

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Note: Figure translations are in progress. See original paper for figures.

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