SCATTERING OF SOUND BY SPIN WAVES

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Abstract

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It is known that in ferromagnets there exists a whole class of nonlinear magnetoacoustic interactions, such as magnetoacoustic resonance ([1]), amplification of sound waves ([2]), etc. This also includes the decay, considered by Morgenthaler ([3]), White and Sparks ([4]), and others, of a spatially homogeneous precession of the magnetization into two acoustic waves. The decay of a homogeneous precession into two phonons arises due to terms in the energy of the ferromagnet that are responsible for the phenomenon of magnetostriction and for the so-called intrinsic effect (intrinsic effect ([5])). These same terms in the energy determine one more type of nonlinear magnetoelastic interaction—the scattering of sound by spin waves. The present work is devoted to a theoretical consideration of such scattering. Combination scattering of sound in uniaxial ferromagnets was recently considered by I. A. Akhiezer and L. N. Davydov in papers ([6]), in which a number of regularities of such scattering were revealed. In these works it was assumed that the scattering is caused by ponderomotive forces and magnetostriction. At the same time, the intrinsic effect plays a large role in the process of combination scattering, and in many cases its contribution is greater than that from magnetostriction. This is connected with the fact that the terms in the expansion of the energy of a ferromagnet responsible for the intrinsic effect are of the same order of smallness in the strains and direction cosines of the magnetic moment as the corresponding magnetostrictive terms; and the constants of the intrinsic effect may be quantities of the same order as, or larger than, the magnetostriction constants. For example, in yttrium iron garnet (YIG), which is of great interest from the point of view of magnetoelastic interactions, some constants of the intrinsic effect exceed the magnetostriction constants in absolute magnitude by more than an order of magnitude ([7]). In the present work the consideration of combination scattering of sound by spin waves is carried out with allowance for the magnetostrictive and intrinsic effects.

Let us consider the practically interesting case of a cubic ferromagnet. We choose the coordinate axes along the natural edges of the crystal. For simplicity we assume that the intense sound wave incident on the medium propagates along the y-axis:

$$\mathbf{u}_0 = \mathbf{u}_0 \frac{\mathbf{k}_0}{|k_0|} \exp i(k_0 y - \omega_0 t),$$

and that the constant internal magnetic field is parallel to the z-axis. The sound wave scattered by fluctuational spin waves is found by solving the nonlinear sound equation by perturbation theory. The nonlinear sound equation is obtained by using the form of the internal energy per unit volume of a cubic ferromagnet ([7]). For the scattered wave $\mathbf{u}$ this equation reduces to

$$\begin{aligned}
&\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} - \left(\tilde{k} + \frac{\mu}{3}\right) \nabla(\nabla \mathbf{u}) - \mu \Delta \mathbf{u} \\
&\quad = u_0 k_0 \exp i(k_0 y - \omega_0 t) \Bigg\{ \mathbf{e}_1 \frac{1}{2} A i \, \frac{\partial \alpha_x(\mathbf{r},t)}{\partial z} \\
&\quad + \mathbf{e}_2 (2b + \tfrac{1}{2} B) i \, \frac{\partial \alpha_y(\mathbf{r},t)}{\partial z} + \mathbf{e}_3 \left[\frac{1}{2} A i \, \frac{\partial \alpha_x(\mathbf{r},t)}{\partial x} \right. \\
&\quad \left. + \left(b + 2a + \tfrac{1}{2} B\right) \left(i \, \frac{\partial \alpha_y(\mathbf{r},t)}{\partial y} - k_0 \alpha_y(\mathbf{r},t) \right) \right] \Bigg\} + \text{c.c.}
\end{aligned} \tag{1}$$

Here $\tilde{k}, \mu$ are elastic moduli; ρ is the density; a, b and A, B are the constants of magnetostriction and of the internal effect, respectively (in the notation of Ref. [7], $A = 2B_{144}, B = 2B_{155}$); α_x, α_y are the direction cosines of the magnetic moment; e_i are unit vectors along the crystal axes.

Equation (1) was obtained under the assumption that the frequencies and wave vectors of the waves under consideration lie far from the intersection point of the dispersion curves, which makes it possible to neglect the linear coupling between sound and spin waves.

The solution of equation (1) in the wave zone for a longitudinal scattered sound wave has the form:

$$\mathbf{u}(\mathbf{r}, t) = \frac{V}{4\pi} \int_{-\infty}^{\infty} \frac{e^{ikr}}{r} \frac{\mathbf{k}\left(\mathbf{k} \cdot \mathbf{f}(\mathbf{k}, \omega)\right)}{k^2 \left(\tilde{k} + \frac{4}{3}\mu\right)} e^{-i\omega t} \, d\omega, \tag{2}$$

where V is the scattering volume; $k = k(\omega)$ is the wave number of the scattered sound; $\mathbf{f}(\mathbf{k}, \omega)$ is the Fourier transform of the right-hand side of equation (1). We do not give here the solution for the transverse scattered wave.

From (2), averaging over fluctuations of the magnetic moment, we obtain the following expression for the ratio γ of the power of sound scattered into the solid angle dO in the frequency interval $d\omega$ to the intensity of the incident sound:

$$\gamma \sim \frac{V}{(4\pi)^2} \frac{k_0^2}{\left(\tilde{k} + \frac{4}{3}\mu\right)^2} \frac{k_z^2}{k^2} \left[A^2 b^2 k_x^2 (\alpha_x)^2_{\mathbf{q},\Omega} + \right.$$

$$+(2a+3b+B)^2 k_0^2 \cos^2\theta \cdot (\alpha_y)^2_{\mathbf{q},\Omega}\Big]\, d\omega\, dO; \quad (3)$$

Here $(\alpha_i)^2_{\mathbf{q},\Omega}$ are the known correlators of magnetic-moment fluctuations (calculated, for example, in [8]); $\mathbf{q}=\mathbf{k}-\mathbf{k}_0$; $\Omega=\omega-\omega_0$; θ is the scattering angle (the angle between $\mathbf{k}_0$ and $\mathbf{k}$).

Let us estimate the contribution of the internal effect in (3) for YIG. Taking, in units of 10^6 erg/cm^3: $a \simeq 3.5$, $b \simeq 7$, $A \simeq -10$, $B \simeq -74$ (see [7]). Since the constants A, B are comparable in absolute value, or even an order of magnitude larger than the magnetostriction constants, allowance for the internal effect, as is evident from (3), gives a value of the intensity of the scattered sound that differs substantially from that which would be obtained by taking only magnetostriction into account. Since, therefore, the intensity of the scattered sound is determined by the constants of the internal effect, an experimental study of combinational scattering of sound in ferromagnets can, in principle, provide information about the magnitude of these constants.

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Note: Figure translations are in progress. See original paper for figures.