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SOME PROBLEMS IN MODELING ULTRALONG ATMOSPHERIC WAVES

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The foundations of the hydrodynamic theory of long-range weather forecasting were laid in 1943 ([1]). At present, a number of numerical schemes have been developed for long-range forecasting with short and long lead times ([2]). In the present article we touch on several questions connected with the possible construction of a numerical scheme for medium-range weather prediction (for 1–3 weeks).

In work ([3]) an attempt was made to explain the peculiarities of the behavior of ultralong waves in the atmosphere by means of the simplest model, including the influence of the "climatic" inhomogeneity of the Earth' s surface. It was shown that the most typical behavior of ultralong waves observed in the atmosphere is possible for the model considered if the ratio of the amplitudes A/a lies in the range from 1 to 1/2. Here A denotes the amplitude of the stationary component of the ultralong wave; a is the amplitude of a particular ultralong wave (which may be intensified by cyclonic waves) at the moment when the phase of its moving component coincides with the phase of the stationary component. The most typical behavior of ultralong waves is oscillations reaching a quarter wavelength to the west and east relative to the normal position.

We do not know an analogous criterion for the real atmosphere. But if, nevertheless, the result obtained in work ([3]) is, with certain assumptions, referred to the real atmosphere, this will mean that the nonstationary component of an ultralong wave may be comparable in amplitude with the stationary component. Hence an ultralong wave may be of interest as an object of forecasting for a shorter period than a season. On the other hand, this means that the nonstationary component is not so large that, in describing ultralong waves, one could neglect the climatic inhomogeneities of the underlying surface. This latter circumstance, in turn, inspires a certain confidence in the possibility of constructing a numerical medium-range forecasting scheme (for 1–3 weeks) which will precompute the planetary circulation of the atmosphere (ultralong waves and the mean zonal flow) as the result of their **nonlinear inertial** interaction with quasi-stationary climatic sources. In this case the influence of disturbances of cyclonic scale can apparently be described parametrically, on the basis of the theory of baroclinic and barotropic instability of the atmosphere.

Before constructing such a forecasting scheme, we must study the principal mechanisms of interaction of the following three processes: 1) cyclonic waves (characteristic time $\tau \sim 10^5$ sec, characteristic horizontal scale $L \sim 10^6$ m), 2) ultralong waves ($\tau \sim 10^6$ sec, $L \sim 10^7$ m), 3) "climate" ($\tau \sim 10^7$ sec, $L \sim 10^7$ m). In a first approximation they may be described by the Phillips model ([4]), supplemented by phenomena,

which depend on inhomogeneities of the underlying surface along a latitude circle:

$$\frac{\partial \nabla^2 \psi^1}{\partial t} - \Lambda^2 \frac{\partial(\psi^1 - \psi^3)}{\partial t} = -(\psi^1, \nabla^2 \psi^1) - \beta \frac{\partial \psi^1}{\partial x} - \Lambda^2 \left[(\psi^1, \psi^3) + \frac{R}{c_p l} \frac{d\theta}{dt} \right]; \quad (1)$$

$$\frac{\partial \nabla^2 \psi^3}{\partial t} + \Lambda^2 \frac{\partial(\psi^1 - \psi^3)}{\partial t} = -(\psi^3, \nabla^2 \psi^3) - \beta \frac{\partial \psi^3}{\partial x} + \Lambda^2 \left[(\psi^1, \psi^3) + \frac{R}{c_p l} \frac{dQ}{dt} \right] - \frac{2gl}{RT_4} \left[w_\xi + w_H - \frac{l}{2g} \frac{\partial \psi^3}{\partial t} \right]. \quad (2)$$

ψ^1 and ψ^3 are stream functions at the levels of 250 and 750 mb;

$$(\psi, \nabla^2 \psi) = \frac{\partial \psi}{\partial x} \frac{\partial \nabla^2 \psi}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial \nabla^2 \psi}{\partial x}; \qquad \nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2;$$

$$\Lambda^2 = \frac{gl^2}{(\gamma_a - \gamma) R^2 T_2} = \text{const}; \qquad \beta = dl/dy$$

(l is the Coriolis parameter); dQ/dt is the vertically averaged radiative heat influx per unit mass; R is the gas constant; c_p is the heat capacity of air at constant pressure; g is the acceleration of gravity; T_2 is the mean air temperature at the 500 mb level; T_4 is the mean air temperature at the 1000 mb level; w_ξ is the vertical velocity at the lower boundary of the atmosphere, generated by the Earth' s relief $\xi(x, y)$:

$$w_\xi = \frac{1}{2}(\psi^3, \xi); \quad (3)$$

w_H is the vertical velocity at the upper boundary of the planetary boundary layer ([5,6]);

$$w_H = \frac{1}{2\sqrt{2l}} \left[\frac{\partial}{\partial x} \left(\frac{\partial \psi^3}{\partial x} - \frac{\partial \psi^3}{\partial y} \right) \sqrt{k} + \frac{\partial}{\partial y} \left(\frac{\partial \psi^3}{\partial x} + \frac{\partial \psi^3}{\partial y} \right) \sqrt{k} \right], \quad (4)$$

$$k = \frac{2G^2}{cl}\left[1 + \frac{2.3gR}{c_p l p_4} - \frac{P}{G^2}\right], \qquad 4G^2 = \left(\frac{\partial \psi^3}{\partial x}\right)^2 + \left(\frac{\partial \psi^3}{\partial y}\right)^2;$$

$P(x, y)$ is the "climatic" turbulent heat flux; $p_4 = 1000$ mb; c is a constant.

We shall seek a particular solution of equations (1)–(2) in the form

$$\psi^j(x, y, t) = -a^j_{00} y + \sum_{n=1}^{N_0} a^j_{2n,0} \sin 2n\lambda y + \sum_{n=1}^{N_1} \sum_{m=1}^{M_1} a^j_{n,m} e^{imkx} \times \sin n\left(\lambda y + \frac{\pi}{2}\right) \quad (5)$$

$$(j = 1, 3), \quad \lambda = \pi/2W, \quad k = 2\pi/L.$$

Thus we reduce (1)–(2) to a system of ordinary differential equations for determining $da^j_{2n,0}/dt$, $da^j_{n,m}/dt$. The boundary conditions in x and y (periodicity in x with period length L, and $v_g \equiv \partial\psi/\partial x = 0$ at $y = \pm W$) will be satisfied automatically if in w_ξ and w_H (for arbitrarily specified $\xi(x, y)$, $P(x, y)$ over land, and an arbitrarily specified ocean surface temperature) only those harmonics are retained which enter into expansion (5). In the present model one may take $a^1_{00} =$ const and $a^3_{00} =$ const ($a^1_{00} > a^3_{00}$) and prescribe the mechanism of radiative heating of the atmosphere in the form [7]:

$$\frac{R}{c_p l}\frac{dQ}{dt} = r \sum_{n=1}^{N_2} \left[(a^{1*}_{2n,0} - a^{3*}_{2n,0}) - (a^1_{2n,0} - a^3_{2n,0})\right] \sin 2n\lambda y, \quad (6)$$

$$\psi^{1*} - \psi^{3*} = \frac{R}{l} T^*_2;$$

T^*_2 is the temperature of radiative equilibrium at the level of 500 mb.

With the aid of (1)–(6) it is important to consider the following problems.

1) The emergence of a nonstationary component of ultralong waves in the numerical integration of equations (1)–(2) over a long time.

To describe more accurately the transfer of kinetic energy to the longest waves from waves of medium scale in the process of nonlinear interaction, and to ensure convergence of the series in x and y, differentiated three times, in the right-hand sides of equations (1)–(2), a sufficiently large number of harmonics must be taken in the expansions of the stream function (5). It is of interest to consider the relationship between the moving and quasistationary (depending on inhomogeneities of the Earth' s surface) components of ultralong waves.

2) To investigate the possibility of precomputing changes in the mean zonal velocity and ultralong waves for a period of 1-3 weeks without an explicit description of cyclonic disturbances, i.e., the possibility of forecasting the Fourier coefficients $a^j_{n,m}(t)$, for example, with indices $m = 0, 1$, and 2, if in the given model $L = 1.5 \cdot 10^7$ m.

Equations (1) and (2) can be written in the form

$$da^j_{n,m}/dt = F^j_{n,m} + f^j_{n,m}, \tag{7}$$

$j = 1, 3;$ $n = 2, 4, 6, \dots, N_0$, if $m = 0;$ $n = 1, 2, 3, \dots, N_1$, if $m = 1, 2, 3, \dots, M_1$, $F^j_{n,m}$ and $f^j_{n,m}$ are certain nonlinear functions of the coefficients $a^1_{n,m}$ and $a^3_{n,m}$, where $F^j_{n,m}$ depend only on $a^1_{00}, a^3_{2,0}, \dots, a^1_{N_0,1};\ a^3_{00}, a^3_{2,0}, \dots, a^3_{N_0,1};\ a^1_{1,1}, \dots, a^1_{N_1,1};\ a^3_{1,1}, \dots, a^3_{N_1,1};\ a^1_{1,2}, \dots, a^1_{N_1,2};\ a$ while $f^j_{n,m}$, generally speaking, depend on all the Fourier coefficients and, most importantly, on the coefficients with indices $m > 2$.

After the full problem has been solved, it is advisable to investigate the accuracy of forecasting the mean zonal velocity and ultralong waves by means of the system of equations (7) for $m = 0, 1$, and 2 only. In this case the functions $f^j_{n,m}$ for $m = 0, 1$, and 2, which include Fourier coefficients with indices $m > 2$, can be expressed through certain combinations and powers of the components of the mean zonal velocity ($m = 0$) and ultralong waves ($m = 1$ and 2). The proportionality coefficients can be found, for example, by the least-squares method for various averaging periods (from one week to three), since from the full problem $f^j_{n,m}$ are known at any moment of time. It is important that, as a result, the proportionality coefficients should be different from zero.

3) After solving the full problem, to investigate the possibility of determining the largest harmonics of a certain "effective" turbulent heat flux from the underlying surface P_{eff} from the dynamical equations through the stream function ψ, known at each moment of time.

Obviously, the mean harmonics of the turbulent heat flux cannot be found in this way, since the quasistationary component of the mean harmonics of the stream function may exceed the nonstationary component (cyclonic disturbances) many times over, and the inverse problem will amount to calculating a small difference of large quantities.

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CITED LITERATURE

1. E. N. Blinova, DAN, 39, No. 7 (1943).

2. E. N. Blinova, Transactions of the Hydrometeorological Center of the USSR, Jubilee Collection, L., 1967.

3. G. P. Kurbatkin, DAN, 177, No. 4 (1967).

4. N. A. Phillips, *Quart. J. Roy. Meteorolog. Soc.*, 82 (1956).

5. S. S. Zilitinkevich, D. L. Laikhtman, Transactions of the Main Geophysical Observatory, issue 167, L., 1965.

6. A. G. Vorob' eva, G. P. Kurbatkin, D. L. Laikhtman, *News of the Academy of Sciences of the USSR, Atmospheric and Oceanic Physics Series*, No. 2 (1967).

7. J. G. Charney, *The Atmosphere and Sea in Motion*, N. Y., 1959.

Note: Figure translations are in progress. See original paper for figures.