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DYNAMICS OF A RIGID ROTOR ROTATING IN ROLLING BEARINGS WITH PRELIMINARY AXIAL PRELOAD

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Radial and angular-contact bearings operating in high-speed assemblies requiring high rotational accuracy (grinding-machine spindles, gyroscopic instruments) have an axial preliminary preload, produced by cylindrical or Belleville springs. An axial load of this kind is most often applied to the outer ring of the ball bearing, increasing the stiffness of the bearing and eliminating the harmful influence of clearances and initial elastic deformations ([1]). In the zone of small preload ([2]) there is a sharp change in the stiffness of the bearing, since, under the action of the centrifugal forces of the balls, the outer spring-loaded ring is displaced and the preliminary preload disappears. Therefore the radial stiffness of the bearing is determined by the stiffness of the preload spring.

1. Let us derive an analytical relation between the stiffness of the axial spring and the radial stiffness of a preloaded ball bearing, assuming that the outer ring of the bearing is free to move in the axial direction (class C bearings of precision spindles are mounted in the housing with guaranteed clearance ([1])).

A dynamic reaction acts on the bearing from the side of the unbalanced rotor. The axis of rotation of the shaft will coincide with the position of static equilibrium until the dynamic component of the shaft reaction exceeds the radial component of the preliminary preload transmitted through the balls from the axial spring. After this the radial stiffness of the support will change sharply, the shaft will begin to execute precessional motion, and the outer ring of the bearing will move in the axial direction until the contact angle of the ball with the inner and outer rings, α (Fig. 1), becomes equal to zero. In this case the stiffness of the bearing will increase practically to infinity (if elastic deformations of the material of the balls and rings at the contact points are not taken into account). Assuming the contact angles of the ball with the outer and inner rings to be identical, we write the relation between the radial and axial clearances of a single-row radial ball bearing ([1]):

$$S = \sqrt{4\delta k - \delta^2},$$

Fig. 1

Figure 1: Fig. 1

where S is the axial clearance; δ is the radial clearance; $k = (R - r)$ is the difference between the groove radius R and the ball radius r; for high-speed bearings $k = 0.03r$.

Let us denote: the coordinate of the radial displacement of the center of the inner bearing ring by z, and the coordinate of the axial displacement of the outer ring by x. Then, from geometrical considerations, we obtain

$$(S - x) = k \sin\alpha; \qquad \cos\alpha = 1 - (\delta - z)/k. \tag{1}$$

The relation between the axial force of the spring and the radial force transmitted through the balls to the shaft is:

$$c_1 x = c_2 z \operatorname{tg}\alpha,$$

where c_1 is the stiffness of the axial spring; c_2 is the stiffness of the bearing in the radial direction; α is the angle of contact of the ball with the ring.

Taking expressions (1) into account, the relation between the axial and radial stiffness characteristics of a radial single-row ball bear-

of a bearing with preload will be

$$c_2 z = \frac{c_1}{4}\left[\sqrt{4k\delta - \delta^2}/\sqrt{4k(\delta - z) - (\delta - z)^2} - 2\right]\left[(2k - \delta) + z\right] + P_0, \tag{2}$$

where $P_0 = A_0 \operatorname{ctg}\alpha_0$ is the radial force due to the preliminary preload; A_0 is the preliminary preload force; α_0 is the initial contact angle, determined by the radial and axial clearances of the bearing.

Figure 2a presents the typical elastic characteristic of such a bearing, and Fig. 2b the calculated elastic characteristic for a radial ball bearing with $D_{\text{in}} = 20$ mm, $\delta = 5 \cdot 10^{-3}$ mm (according to the standard series) and $c_1 = 100$ kgf/cm.

2. Let us consider the dynamics of a rigid rotor rotating in one support, which is a radial ball bearing with preload, and a second ball support. Let B be the equatorial moment of inertia of the rotor with respect to an axis passing through the ball support; l the length of the rotor between supports. The elastic characteristic of the ball bearing is determined by expression (2). Using Lagrange' s equations, we obtain nonlinear differential equations of motion for an unbalanced rotor with unbalance Me (where M is the mass of the rotor, e is the displacement of the rotor' s center of gravity from the geometric axis), without taking into account gyroscopic forces and damping:

Fig. 2

Figure 2: Fig. 2

Fig. 1. Calculation scheme of the rotor and bearing assembly with preliminary preload

Fig. 2. Elastic characteristic of a radial single-row ball bearing with preliminary preload:
a –principal scheme, b –calculated curve

$$P(z) = 5\left[\frac{2.96}{\sqrt{1.8(5-z)-(5-z)^2\cdot 10^{-2}}}-2\right](0.085+z\cdot 10^{-3})+7.2$$

$$\ddot{z}+p^2\left[S/\sqrt{4k(\delta-z)-(\delta-z)^2}-2\right](\theta+z)+q_0\sin\omega t+h\omega^2\sin\omega t, \quad (3)$$

$$\ddot{y}+p^2\left[S/\sqrt{4k(\delta-y)-(\delta-y)^2}-2\right](\theta+y)+q_0\cos\omega t=h\omega^2\cos\omega t. \quad (4)$$

Here

$$p^2 := \frac{c_1}{4}\frac{l^2}{B}; \qquad \theta=(2k-\delta); \qquad q_0+P_0l^2/B; \qquad h=Mel^2/B,$$

ω is the angular velocity of rotation of the rotor.

To find periodic solutions of equations (3), (4), we use Galerkin' s variational method ([3]). In the first approximation we shall seek these solutions in the form: $z=a\sin\omega t$, $y=b\cos\omega t$.

Then the basic relation allowing one to find the dependence between the amplitude and the frequency of oscillations will take the form ([4]): for the equation

$$\mathfrak{a}(0)-a\omega^2=h\omega^2 \quad (5)$$

and for equation (4)

$$\beta(b)-b\omega_0^2=h\omega^2. \quad (6)$$

The problem reduces to finding the Fourier integrals

$$\alpha(a)=\frac{1}{\pi}\int_0^{2\pi} f_1(a\sin\omega t)\sin\omega t\,d\omega t, \quad (7)$$

$$\beta(b) = \frac{1}{\pi}\int_0^{2\pi} f_2(b\cos\omega t)\cos\omega t\, d\omega t, \tag{8}$$

where

$$f_1(a\sin\omega t) = p^2\left[S\sqrt{4k(\delta - a\sin\omega t) - (\delta - a\sin\omega t)^2} - 2\right](\theta + a\sin\omega t) + q_0\sin\omega t,$$

$$f_2(b\cos\omega t) = p^2\left[S\sqrt{4k(\delta - b\cos\omega t) - (\delta - b\cos\omega t)^2} - 2\right](\theta + b\cos\omega t) + q_0\cos\omega t.$$

The elastic characteristic of the bearing with respect to the coordinate y is analogous to the elastic characteristic with respect to the coordinate z. Therefore, calculation of the integrals (7) and (8) gives identical expressions for $\alpha(a)$ and $\beta(b)$; and since, owing to the absence of gyroscopic forces, equations (3) and (4) are not coupled, it is sufficient to solve one of these equations.

To solve equation (3), we find the integral (7), which decomposes into three integrals:

a)

$$S\frac{p^2}{\pi}\int_0^{2\pi}\frac{(\theta + a\sin\omega t)\sin\omega t\, d\omega t}{\sqrt{4k(\delta - a\sin\omega t) - (\delta - a\sin\omega t)^2}};$$

b)

$$2\frac{p^2}{\pi}\int_0^{2\pi}(\theta + a\sin\omega t)\sin\omega t\, d\omega t;$$

c)

$$\frac{q_0}{\pi}\int_0^{2\pi}\sin^2\omega t\, d\omega t.$$

To find integral a), the function

$$f_1(z) = \frac{\theta + z}{\{2k(\delta - k) - (\delta - z)^2\}^{1/2}}$$

is expanded in a power series in z. To ensure the necessary accuracy of the calculations, terms of the series up to the fourth power of z were taken. The integral (7) found is equal to

$$\alpha(a) = p^2 S(B + C\theta)a + p^2 S(D + E\theta)^{3/4} a^3 - 2p^2 a + q_0; \qquad (9)$$

the coefficients $S(B + C\theta)$ and $S(D + E\theta)$ are determined by the formulas:

$$S(B + C\theta) = \frac{(\lambda - 1)^2}{(2\lambda - 1)}, \qquad (10)$$

$$S(D + E\theta)^{3/4} = \gamma = \frac{1}{\delta^2}\frac{0.375}{(2\lambda - 1)}\left[\frac{3(\lambda - 1)^2}{(2\lambda - 1)} + \frac{5(\lambda - 1)^4}{(2\lambda - 1)^2} + 1\right], \qquad (11)$$

where $\lambda = 2k/\delta$.

Substituting the expression obtained for $\alpha(0)$ into equation (5), we find the dependence between the amplitude and the frequency of rotor oscillations:

$$\Omega^2 = \frac{(\lambda - 1)^2}{(2\lambda - 1)} - 1 + \gamma\xi^2 + \nu\frac{1}{\xi} - h\frac{1}{\xi}\Omega^2. \qquad (12)$$

Here the dimensionless parameters have been introduced

$$\Omega^2 = \omega^2/p^2; \qquad \nu = q_0/\delta p^2 = 4P_0/\delta c_1; \qquad \eta = h/\delta; \qquad \xi = a/\delta.$$

In the case of free oscillations, equation (12) takes the form

$$\Omega^2 = \frac{(\lambda - 1)^2}{(2\lambda - 1)} - 1 + \gamma\xi^2 + \nu\frac{1}{\xi}. \qquad (13)$$

Expression (13) is the equation of the backbone curves. In Fig. 3 the backbone lines calculated for $\nu = 0;\ 0.5;$ and 5 are presented.

With an increase in the dimensionless preload v, the character of curve (13) changes, approaching an inversely proportional dependence. The same will occur with an increase in λ (a decrease in the radial clearance δ). Fig. 4 presents amplitude-frequency characteristics calculated for $\lambda = 1$, $v = 50$ and for three values of the dimensionless unbalance: $\eta = 0.1;\ 0.2;\ 0.3$. It is seen from the figure that, for certain values of η (in the present case $\eta < 0.3$), passage through the resonance zone without a significant increase in the vibration amplitude is possible, since during acceleration there will be a jump of the amplitude from the lower branch to the upper one. With an unbounded increase in the dimensionless frequency Ω, the amplitude tends to a certain limit defined by the expression:

$$\lim_{\Omega^2 \to \infty} \xi = -\eta \quad \text{(Fig. 4)}$$

or

$$a_{\omega\to\infty} = -Mel^2/B. \quad (14)$$

Fig. 3. Skeleton curve of the amplitude-frequency characteristics of vibrations of a rigid rotor in a bearing with preload: $17+8.6\xi^2 = \Omega^2$ for $v=0$; $17+8.6\xi^2+0.5/\xi = \Omega^2$ for $v=0.5$; $17+8.6\xi^2+5/\xi = \Omega^2$ for $v=5$

Fig. 4. Amplitude-frequency characteristics of a rigid rotor ($\lambda = 1$, $v = 50$). Curve A at $\eta = 0$ corresponds to the equation $\Omega^2 = 0.5\xi^2 + 50/\xi$

At run-out, capture into the region of low frequencies is possible; however, by an appropriate selection of the magnitude of the preliminary preload (by reducing it), a significant increase in amplitude can be avoided (Fig. 3). In addition, the absence of capture into the region of low frequencies has been experimentally demonstrated for a system with a soft characteristic. Consequently, preliminary preload in rolling bearings can play the role of a vibration regulator, allowing the stiffness of the support to be varied.

Thus, a ball bearing with preliminary axial preload can operate similarly to a dry-friction damper (5), limiting vibration amplitudes in the resonance zone, while centering of the rotor at super-resonance rotational speeds is guaranteed. The mode of passage through the resonance zone must be selected in accordance with the magnitude of the rotor unbalance and depends on the preload parameter $v = 4P_0/\delta c_1$. The most optimal variant from the standpoint of the rotor' s dynamic characteristics will apparently be a combination of an elastic support (6) with a ball bearing having axial preliminary preload.

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REFERENCES CITED

1. A. I. Sprishevsky, *Rolling Bearings*, Moscow, 1969.
2. A. S. Saversky, *Transactions of the All-Union Scientific Research Institute of the Bearing Industry*, No. 3 (1960).
3. A. I. Lur' e, A. I. Tsekmariev, *PMM*, new series, 1, issue 3 (1938).
4. I. I. Vulfson, M. Z. Kolovsky, *Nonlinear Problems in Machine Dynamics*, Leningrad, 1968.
5. E. L. Poznyak, in: *Machine Dynamics*, Moscow, 1969.

6. A. S. Kel' zon, *DAN*, 110, No. 1 (1956).

Note: Figure translations are in progress. See original paper for figures.