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ESTIMATING THE CLOSENESS OF FUNCTIONS OF BOUNDED VARIATION FROM THE CLOSENESS OF THEIR FOURIER-STIELTJES TRANSFORMS

MATHEMATICS

1970

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Abstract

Full Text

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MATHEMATICS

V. V. PETROV

ESTIMATING THE CLOSENESS OF FUNCTIONS OF BOUNDED VARIATION FROM THE CLOSENESS OF THEIR FOURIER-STIELTJES TRANSFORMS

(Presented by Academician Yu. V. Linnik on 1 XII 1969)

An important role in probability theory is played by Esseen' s inequalities ([1,2]), which make it possible to estimate the closeness of a nondecreasing bounded function $F(x)$ and a function of bounded variation $G(x)$ from the closeness of the corresponding Fourier-Stieltjes transforms on some finite interval. It is assumed here that either $G(x)$ has a derivative uniformly bounded in x, or $F(x)$ is a purely discontinuous function such that the functions $F(x)$ and $G(x)$ may have discontinuities only at points $x = x_\nu$ $(x_\nu < x_{\nu+1}, \nu = 0, \pm 1, \pm 2, ...)$, satisfying the condition $\min_\nu (x_{\nu+1} - x_\nu) \geq L > 0$, with $|G'(x)| \leq A$ everywhere, except at the points $x = x_\nu$. These results of Esseen can be encompassed by a single formulation. The following theorem is a generalization of Esseen' s theorems.

Theorem 1. Let $F(x)$ be a nondecreasing function, and $G(x)$ a function of bounded variation on the real line, $F(-\infty) = G(-\infty)$, $F(+\infty) = G(+\infty)$. Let

$$f(t) = \int_{-\infty}^{\infty} e^{itx}\, dF(x), \qquad g(t) = \int_{-\infty}^{\infty} e^{itx}\, dG(x),$$

and let T be an arbitrary positive number. Then, for any number $b > 1/2\pi$, the inequality

$$\sup_x |F(x) - G(x)| \leq b \int_{-T}^{T} \left| \frac{f(t) - g(t)}{t} \right| dt + bT \sup_x \int_{|y| \leq c(b)/T} |G(x+y) - G(x)|\, dy, \tag{1}$$

holds, where $c(b)$ is a positive constant depending only on b.

In inequality (1) one may take $c(b)$ equal to the root of the equation

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$$\int_0^{1/4\,c(b)} \frac{\sin^2 u}{u^2}\, du = \frac{\pi}{4} + \frac{1}{8b}.$$

In the special case when $F(x)$ and $G(x)$ are distribution functions, a similar result was obtained by A. C. Feinleib ([3]). A uniform estimate of the difference between a distribution function $F(x)$ and a certain function of bounded variation $G(x)$ that is not a distribution function is of considerable interest for applications (for example, in the study of asymptotic expansions in limit theorems for sums of independent random variables).

If the conditions of Theorem 1 are satisfied and the function $G(x)$ satisfies the following Lipschitz condition:

$$|G(x) - G(y)| \le K|x - y|^\alpha$$

for all x and y and for some positive constants K and α, then the second term on the right-hand side of inequality (1) may be replaced by $2bK(c(b))^{1+\alpha}(1 + \alpha)^{-1}T^{-\alpha}$.

We give one more immediate consequence of Theorem 1, which is also a generalization of Esseen' s theorem.

Theorem 2. Let $F(x)$ be a nondecreasing function, $G(x)$ a function of bounded variation, and let $f(t)$ and $g(t)$ be the corresponding Fourier-Stieltjes transforms; let T be an arbitrary positive number, and

$$F(-\infty) = G(-\infty), \qquad F(+\infty) = G(+\infty).$$

Suppose $|G'(x)| \le A$ everywhere, except at points of discontinuity of the function $G(x)$.

Then, for any number $b > 1/2\pi$, the inequality

$$\sup_x |F(x) - G(x)| \le b \int_{-T}^{T} \left| \frac{f(t) - g(t)}{t} \right| dt + r(b)\frac{A}{T}, \tag{2}$$

holds, where $r(b)$ is a positive constant depending only on b.

In inequality (2) one may take $r(b) = bc^2(b)$, where $c(b)$ is the constant from Theorem 1.

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Note: Figure translations are in progress. See original paper for figures.

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