THE ORIGIN AND DEVELOPMENT OF THERMAL CONVECTION IN A LAYER OF VISCOUS LIQUID

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Abstract

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THE ORIGIN AND DEVELOPMENT OF THERMAL CONVECTION IN A LAYER OF VISCOUS LIQUID

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Most works on thermal convection are devoted to the study either of stationary regimes or of the stability of a layer in a field of body forces (see, for example, ($^{1-5}$)). The nonstationary aspects of convective motion and its influence on heat transfer have been studied poorly. In the present work, on the basis of measurements of a nonstationary temperature field, the process of the origin of thermal convection in a plane horizontal layer of viscous liquid is investigated.

The experimental procedure is shown in Fig. 1. The liquid under investigation, at room temperature T_k, was poured into cuvette 2, which was an aluminum plate 1 mm thick with side walls made of organic glass glued to it. The dimensions of the cuvette were 23 × 19 × 4 cm. The surface of the liquid was in contact with air at temperature T_k. The cuvette was placed on heat exchanger 1. Temperature measurement was carried out with a differential copper-constantan thermocouple (junction diameter 80μ), one junction of which was at the wet-bulb temperature of the liquid, and the other in the liquid layer at a specified distance from the bottom of the cuvette. The nonstationary temperature profile in the liquid layer under specified experimental conditions was obtained as the result of carrying out a series of identi-

Fig. 1. Schematic of the apparatus. 1 —heat exchanger; 2 —cuvette; 3 —layer of the liquid under investigation; 4 —device with a movable thermocouple; 5 —fixed thermocouple; 6 —control thermocouples

Table 1

Liquid	Water content, %	v_0, cm^2/sec	u, cal/mole	$\alpha \cdot 10^4$, cm/sec	$\lambda \cdot 10^4$, cal/(cm·sec·deg)	$\beta \cdot 10^4$, 1/deg	T_0, °C	h, cm
MS-20 type oil		$3.6 \cdot 10^{-10}$	14200	7.1	3.2	6.35	30–60	0.5–4
Aqueous glycerin solution with water content (%)	0	$7.7 \cdot 10^{-6}$	8360	9.45	6.67	4.7	20–50	0.5–4
Aqueous glycerin solution with water content (%)	10	$7.7 \cdot 10^{-5}$	5900	9.7	7.48	4.5	20–50	0.5–4
Aqueous glycerin solution with water content (%)	20	$3.5 \cdot 10^{-5}$	5580	9.94	8.3	4.3	20–40	0.5–3

Liquid	Water content, %	v_0, cm^2/sec	u, cal/mole	$\alpha \cdot 10^4$, cm/sec	$\lambda \cdot 10^4$, cal/(cm·sec·deg)	$\beta \cdot 10^4$, 1/deg	T_0, °C	h, cm
Aqueous glycerin solution with water content (%)	30	$1.33 \cdot 10^{-5}$	5580	10.1	9.05	4.1	20–35	0.5–3
Aqueous glycerin solution with water content (%)	40	$2.65 \cdot 10^{-6}$	6100	10.4	9.9	3.9	20–30	0.5–2
Benzyl alcohol		$7.75 \cdot 10^{-5}$	3700	6.6	3.75	1.18	20–40	0.5–3
Isobutyl alcohol		$7.2 \cdot 10^{-6}$	5180	6.1	3.26	1.4	20–30	0.5–3
Water		$1.1 \cdot 10^{-5}$	3990	15.2	14.65	2.1	20–30	0.5–2

of identical experiments differing only in the position of the thermocouple junction 5, which was possible owing to the good reproducibility of the experimental data. In some experiments, in order to obtain rapid information on the temperature profile, a movable thermocouple 4 was used, whose junction made periodic displacements in the liquid layer at a constant speed.

The experiments were carried out with different liquids. The liquid constants and the experimental conditions are given in Table 1. The viscosity of all substances was measured with a capillary viscometer at various temperatures. The remaining constants were taken from handbooks (6, 7).

Fig. 2

Figure 2: Fig. 2

Fig. 2. Unsteady temperature pattern in a liquid layer;
$\mathrm{Ra} = 1.76 \cdot 10^4$; $\mathrm{Pr} = 10^4$; solid lines $-\theta$; dashed lines $-\hat{\theta}$.
$a:\ \xi = 0.2\ (1),\ 0.25\ (2),\ 0.35\ (3),\ 0.5\ (4),\ 0.6\ (5),\ 0.8\ (6)$.
$b:\ \tau = 3.5 \cdot 10^{-2}\ (1),\ 4 \cdot 10^{-2}\ (2),\ 4.5 \cdot 10^{-2}\ (3),\ 6 \cdot 10^{-2}\ (4),\ \tau \to \infty\ (5)$.

The process under investigation began at the moment the cuvette and the heat exchanger were brought into contact. In this case it could be assumed that at time $t = 0$ the temperature of the liquid layer and of the air above it was T_k, while the temperature of the liquid at the bottom of the cuvette was T_0. The result of the experiment was the determination of the form of the dependence

$$T = T(x, t, T_0, h, b_i),$$

where b_i are the constants of the liquid.

The experimental data were processed in dimensionless quantities. The variables were: $\theta = (T - T_k)/(T_0 - T_k)$; $\xi = x/h$; $\tau = at/h^2$. The parameters were: $\mathrm{Ra} = g\beta(T_0 - T_k)h^3/a\nu(T_0)$ —the Rayleigh number; $\mathrm{Pr} = \nu/a$ —the Prandtl criterion; $\mathrm{Bi} = \alpha h/\lambda$ —the Biot criterion.

Notation: x —distance of the thermocouple from the bottom of the cuvette (cm); h —height of the liquid layer (cm); ν —coefficient of kinematic viscosity (cm^2/sec); a —coefficient of thermal diffusivity (cm^2/sec); λ —coefficient of thermal conductivity (cal/cm $\cdot$ sec $\cdot$ deg); α —heat-transfer coefficient (cal/cm^2 $\cdot$ sec $\cdot$ deg); g —acceleration of gravity (cm/sec^2); β —coefficient of volume expansion (1/deg).

The heat-transfer coefficient from the surface of the layer into the air was, according to (9), $\sim 10^{-4}$ (cal/cm^2 $\cdot$ sec $\cdot$ deg). The possibility of using the concept of a heat-transfer coefficient for the given unsteady process is due to the fact that the time of thermal relaxation processes in the gas is much smaller than in the liquid layer.

The influence of convection on the unsteady temperature field was characterized by the value of the function

$$\vartheta = \theta(\xi, \tau, \mathrm{Ra}, \mathrm{Pr}, \mathrm{Bi}) - \hat{\theta}(\xi, \tau, \mathrm{Bi}),$$

where $\hat{\theta}$ is the solution of the problem of unsteady heat conduction in a plane wall with the boundary and initial conditions (8)

$$\xi = 0,\ \theta = 1; \qquad \xi = 1,\ -\partial\theta/\partial\xi = \mathrm{Bi}\,\theta;$$

Fig. 3. Dependence of ϑ_m on the number Ra at $\mathrm{Pr} = 10^4$ (a), 7900 (b), 950 (c)

Figure 3: Fig. 3. Dependence of ϑ_m on the number Ra at $\mathrm{Pr} = 10^4$ (a), 7900 (b), 950 (c)

$$\tau = 0, \quad \theta = 0.$$

A typical picture of the unsteady temperature field of the liquid layer, obtained on the basis of experimental data, is shown in Fig. 2. The principal result of the experiment, as is seen from this figure, is the existence of an induction period: convective motion throughout the entire liquid layer and its influence on heat transfer arise not immediately, but after a certain induction period τ_{ind}. For $\tau < \tau_{\text{ind}}$ the motion of the liquid in the layer is insignificant (or absent), and heat transfer proceeds according to the laws of unsteady heat conduction. For $\tau > \tau_{\text{ind}}$ the arising motion of the liquid sharply intensifies heat exchange and distorts the temperature profile. Since the values of τ_{ind} for different ξ are approximately the same, one may speak of a single induction period for the entire layer. The quantity $\varepsilon = \Delta\tau/\tau_{\text{ind}}$ reflects the character of the onset of convection. The smaller ε, the more sharply expressed is the induction mechanism of the onset of convection.

Fig. 3. Dependence of ϑ_m on the number Ra at $\mathrm{Pr} = 10^4$ (a), 7900 (b), 950 (c)

The picture of the onset of convection depends substantially on Ra. Figure 3 presents the dependence of the quantity ϑ_m on Ra and Pr at $\xi = 0.7$. The transition from the conductive regime to the convective heat-transfer regime in the layer as Ra increases occurs rather sharply, so that one may speak of critical conditions for the onset of convection and characterize its influence by the quantity Ra_{cr}. The method for determining Ra_{cr} is clear from Fig. 3. For the liquids investigated ($\mathrm{Pr} > 10^2$), Ra_{cr} does not depend on Pr and is equal to 1150.

Neither Ra_{cr} nor τ_{ind} depends appreciably on Bi. This is understandable, since the quantity Bi should influence not the onset of convection, but the establishment of the steady convective regime.

The dependence of τ_{ind} on Ra and Pr, obtained as a result of processing a large amount of experimental data, is presented in Fig. 4. For $\mathrm{Ra} > 10^4$ and $\mathrm{Pr} > 10$ this dependence can be described by the empirical formula

$$\tau_{\text{ind}} = A\mathrm{Ra}^m\mathrm{Pr}^n,$$

where $A = 70$, $n = -1/6$, $m = -2/3$.

In dimensional quantities this formula has the form

Fig. 4. Logarithmic dependence of the induction period on the number Ra at Pr = 10^4 (a), 7900 ( ), 950 ( ), 278 ( ), 66 ( ), 54 ( ), 6.8 ( )

Figure 4: Fig. 4. Logarithmic dependence of the induction period on the number Ra at Pr = 10^4 (a), 7900 ( ), 950 ( ), 278 ( ), 66 ( ), 54 ( ), 6.8 ( )

$$t_{\text{ind}} = A\left[g\beta(T_0 - T_k)\right]^{-2/3} \nu^{5/6} a^{-1/6}.$$

It is characteristic that, at some distance from the limit of the onset of convection, the induction period ceases to depend on the thickness of the liquid layer.

In some experiments, simultaneously with the temperature measurements, visual observations were carried out of the process in the liquid, to which light-scattering aluminum particles of size $5 \div 10\,\mu$ had been added.

Observations showed that for a certain interval of time after the start of heating the liquid remained undisturbed. Then, at an arbitrary point near the bottom of the cuvette, perturbations of a convex, dome-shaped form arose and gradually spread over the entire horizontal plane, after which they rapidly reached the free surface of the liquid. It is precisely this instant of time that corresponded to the value τ_{ind} determined from the temperature curves. Subsequently, the ascending and descending flows that formed produced an ordered structure.

Fig. 4. Logarithmic dependence of the induction period on the number Ra at Pr = 10^4 (a), 7900 (b), 950 (c), 278 (d), 66 (e), 54 (f), 6.8 (g)

It is interesting to note that between the processes of the onset of thermal convection and thermal explosion there is a formal analogy, which left its imprint on the experimental procedure and made it possible to use such well-known characteristics as the critical conditions and the induction period.

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Note: Figure translations are in progress. See original paper for figures.