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LIMIT EQUILIBRIUM OF REINFORCED SHELLS ON A RIGID-PLASTIC FOUNDATION

MECHANICS

1970

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

UDC 531.2

MECHANICS

N. V. AKHVELDIANI

LIMIT EQUILIBRIUM OF REINFORCED SHELLS ON A RIGID-PLASTIC FOUNDATION

(Presented by Academician Yu. N. Rabotnov, 23 IX 1969)

1. Let us consider a reinforced shell for which the projections of two horizontal sections of the middle surface are similar m-gons with vertices $a, b, c, \ldots$ and $A, B, C, \ldots$, respectively, and such that

$$Oa/OA = Ob/OB = Oc/OC = \ldots, \tag{1}$$

where O is a point inside the projection of the shell (Fig. 1).

For $m = \infty$, similar polygons become similar curvilinear figures, and the design relations are obtained from those given below by a limiting transition.

The support conditions are taken in the form of vertical and horizontal rigid-plastic support rods, uniformly distributed along the horizontal contours satisfying relations (1). Rigid fixing is considered as a particular case of rigid-plastic fixing, in which the limiting magnitude of the reaction is equal to infinity.

It is assumed that the hodographs of the materials and of the load are similar to the middle surface, and that the fracture pattern in the state of limit equilibrium is a set of horizontal piecewise-linear plastic hinges joined at the corners by through cracks.

Fig. 1

Let us write the equation of virtual work of the external and internal forces, including the reactions of the rigid-plastic foundation among the external forces. The level of location of the axes of mutual rotation of the links will be chosen taking into account the considerations set forth in [1].

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Fig. 2

Figure 2: Fig. 2

Introduce the following notation: t is the number of the horizontal row of links of the plastic mechanism; s is the number of a link in the t-th row, counted from the initial half-plane common to all t; $w'_{ti}, w''_{ti}, u'_{ti}, u''_{ti}$ are the intensities of the vertical and horizontal components of the load and of the reactions of the rigid-plastic foundation, uniformly distributed along the contours of the i-th horizontal sections; δ_{ti}, τ^s_{ti} are the corresponding vertical and horizontal displacements; $l_{t,t+1}$ is the distance from the point O to the vertical plane containing the axis of the part of the hinge that separates the links (t, s) and $(t+1, s)$; h^s_{ti} is the analogous quantity for the contour of the i-th section; $m_{t,t+1}$ is the bending moment per unit length in the hinge $(t, t+1)$; $\varphi_{t,t+1}$ is the corresponding angle of fracture; φ^s_t is the angle of rotation in space of the link (t, s); Z_{ti} is the projection of the limiting force in the rod intersecting, at the i-th level, the plane of the corner crack onto the normal to this plane; d_{ti} is the distance

from the plane of the i-th level to the plane of the axes of rotation of the members of the corresponding row; α^s, β^s are the dihedral angles shown in Fig. 2.

From condition (1) it follows that

$$h^s_{ti}/h^0_{ti} = h^s_{t,t+1}/h^0_{t,t+1} = \varphi^0_t/\varphi^s_t = \tau^0_{ti}/\tau^s_{ti} = \lambda^s, \tag{2}$$

where λ^s is a number constant for each s.

Taking (2) into account, we write the expression for the virtual work of the running moments in the form

$$A \sum^t m_{t,t+1}\, h^0_{t,t+1} \varphi^0_{t,t+1}, \tag{3}$$

where

$$A = \sum^s (\operatorname{tg} \alpha^s + \operatorname{tg} \beta^s). \tag{4}$$

The expression for the virtual work in the angular cracks is obtained by multiplying the quantities Z_{ti}, d_{ti} and the sum of the fracture angles in the angular cracks. Taking (2) into account, we obtain:

Fig. 2

$$B \sum^t \varphi^0_t \sum^i Z_{ti} d_{ti}, \tag{5}$$

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where

$$B = \sum^{s} \frac{1}{\lambda^s}(\sin\alpha^s + \sin\beta^s). \tag{6}$$

The virtual work of the external forces and reactions in the plastically deforming rods of the rigid-plastic foundation is computed analogously, taking (2) into account,

$$A\sum^{t}\sum^{i} h^0_{ti}u_{ti}\tau^0_{ti} + C\sum^{t}\sum^{i} h^0_{ti}w_{ti}\delta_{ti}, \tag{7}$$

where

$$u_{ti} = u'_{ti} + u''_{ti}, \qquad w_{ti} = w'_{ti} + w''_{ti}; \tag{8}$$

$$C = \sum^{s} \lambda^s(\operatorname{tg}\alpha^s + \operatorname{tg}\beta^s). \tag{9}$$

Equating the sum of expressions (3) and (5) to expression (7) and dividing all terms of the equation by A, we obtain the equation of virtual work of the system

$$\sum^{t}\sum^{i} h^0_{ti}u_{ti}\tau^0_{ti} + K_1\sum^{t}\sum^{i} h^0_{ti}w_{ti}\delta_{ti} = \sum^{t} m_{t,t+1}h^0_{t,t+1}\varphi^0_{t,t+1} + K_2\sum^{t}\varphi^0_t\sum^{i} Z_{ti}d_{ti}, \tag{10}$$

where $K_1 = C/A$; $K_2 = B/A$.

2. Taking into account that

$$\varphi^0_t d_{ti} = \tau^0_{ti}, \tag{11}$$

one can write the algebraic sum of the first and last terms of equation (2) in the form

$$\sum^{t}\sum^{i}(h^0_{ti}u_{ti} - K_2 Z_{ti})\,\tau^0_{ti}. \tag{12}$$

After such a substitution, expression (2) can be interpreted as the equation of limiting equilibrium of a certain plane reinforced el-

element (arch) with limiting moments $m_{t,t+1}h^0_{t,t+1}$, loaded by vertical forces $K_1 w_{ti}h^0_{ti}$ and horizontal forces $h^0_{ti}u_{ti} - K_2 Z_{ti}$, which include the reactions both of the actual and of a certain fictitious rigid-plastic foundation.

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The reduction of the spatial problem to a plane one simplifies the numerical solution, in particular programs for electronic digital computers, and makes it possible to use for spatial problems two-sided estimates of the bearing capacity intended for plane systems—arches ([2]).

3. For a prescribed outline of the axis of the above-mentioned computational plane element (arch), one can always select such a system of forces acting on it (including the reactions of the rigid-plastic foundation) that the prescribed axis would be a funicular curve. Let us distinguish such forces in (2), marking them with an asterisk, and choose the centers of mutual rotation of the links on the given geometric axis. Taking (3) into account, we obtain:

$$\sum^{t}\sum^{i}\left(h^0_{ti}u^*_{ti} - K_2 Z^*_{ti}\right)\tau^0_{ti} + K_1\sum^{t}\sum^{i}h^0_{ti}w^*_{ti}\delta_{ti} + \sum^{t}\sum^{i}\left(h^0_{ti}u_{ti} - K_2 Z_{ti}\right)\tau^0_{ti} + K_1\sum^{t}\sum^{i}h^0_{ti}w_{ti}\delta_{ti} = \sum^{t} m_{t,t+1} \qquad (13)$$

Since the forces marked with an asterisk constitute an equilibrated system of forces acting on the given pin-bar polygon (moment-free equilibrium), the sum of the first two terms in (4) is equal to zero. Consequently, when determining the limiting values of the parameters of the remaining forces (which are in a state of moment equilibrium), one may ignore the components marked with an asterisk and construct the complete solution as the sum of the partial solutions obtained for moment-free and moment equilibrium states. Such a superposition of solutions may be interpreted as a restricted application of the superposition principle in the theory of limit equilibrium of reinforced shells.

Institute of Construction Mechanics and Earthquake Resistance Academy of Sciences of the Georgian SSR Tbilisi

Received 17 IX 1969

CITED LITERATURE

[1] N. V. Akhvlediani, *Proceedings of the International Conference on Continuum Mechanics*, Varna, 1966.
[2] N. V. Akhvlediani, *Construction Mechanics and Analysis of Structures*, No. 2 (1960).

Note: Figure translations are in progress. See original paper for figures.

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