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SELF-SIMILAR WAVES IN A COLD PLASMA WITH A MAGNETIC FIELD

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Nonstationary waves in a plasma with a magnetic field

The following problems are solved: at time $t = 0$ there is a cylindrical plasma filament or a plane plasma layer with density decreasing with distance from the axis as n_0/r^2, and with electric and magnetic fields decreasing as E_0/r, H_0/r, or a spherical plasma cloud (in the spherical case–without a magnetic field, but with an electric field E_0/r at $t = 0$). Determine the motion of the plasma and the field for $t > 0$. The substitution

$$E_\phi = \frac{1}{r}E(\eta); \quad H_\phi = \frac{1}{r}H(\eta); \quad v_\phi = v(\eta); \quad n_\phi = \frac{1}{r^2}n(\eta); \quad \eta = t/r \quad (1)$$

(the subscript ϕ denotes physical field vectors) makes it possible to reduce Maxwell' s equations and the equations of two-fluid hydrodynamics

$$\operatorname{rot} H_\phi = j_\phi + \partial E_\phi/\partial t; \quad \operatorname{rot} E_\phi = -\partial H_\phi/\partial t; \quad \partial p_{i,e}/\partial t + (v_{i,e}\nabla)p_{i,e} =$$

$$= e\left(E + [v_{i,e}H]\right); \quad j_\phi = e(n_i v_i - n_e v_e); \quad p_{i,e} = mv_{i,e}/\sqrt{1 - v_{i,e}^2}. \quad (2)$$

to a system of ordinary differential equations (*). Below, the results of solving this system on a machine are presented.

Interchanging t and r in (1), we obtain a system of equations with independent variable $\xi = r/t$, describing expansion for a prescribed regime at the origin of coordinates.

In the nonrelativistic approximation there exist two limiting classes of self-similar solutions (s.s.s.) of system (1). In solutions of the longitudinal-magnetic type (LM), $H_z, E_\phi, u_\phi, v_\phi, u_r, v_r$ are nonzero (v is the electron velocity, u the ion velocity). In solutions of the longitudinal-electric type (LE), $E_z, H_\phi, u_z, v_z, u_r, v_r$ are nonzero. Mixed initial conditions and relativistic terms $v(Ev)$ lead to more complicated s.s.s.

Let us begin with the limiting case $n_i = n_e = 0$, with s.s.s. in vacuum. Then equations (2) have exact analytical s.s.s.:

$$H_z = H_{z0}/\sqrt{1-\eta^2}; \quad E_\phi = E_{\phi 0} + \eta H_{z0}/\sqrt{1-\eta^2}; \quad E_z = E_{z0}/\sqrt{1-\eta^2};$$

$$H_\phi = H_{\phi 0} - \eta E_{z0}/\sqrt{1-\eta^2}. \tag{3}$$

The solution with E_z, H_ϕ describes the process of decrease of the electromagnetic field upon sudden termination of the axial current j_z. In the plane case:

$$H_z = (H_{z0} + \eta E_{y0})/(1-\eta^2); \quad E_y = (E_{y0} + \eta H_{z0})/(1-\eta^2). \tag{4}$$

The invariant

$$E^2 - H^2 = \frac{(E_{y0}^2 - H_{z0}^2)}{x^2 - t^2} = (E_{y0}^2 - H_{z0}^2)\frac{1}{2t}\left[\frac{1}{x+t} - \frac{1}{x-t}\right]$$

is a function only of the other invariant of Lorentz transformations—the interval $x^2 - t^2$.

A self-similar solution can also be obtained by the substitution $E, H = r^\beta f(\eta)$ (β arbitrary), but only for $\beta = -1$ are the Poisson equations and the equations of motion of the plasma also self-similar. A self-similar solution of type (4) can be represented in the form of a superposition of two wave processes:

$$H_{z\phi} = \frac{H_{z0}}{2}\left[\frac{1+Q}{x-t} + \frac{1-Q}{x+t}\right];$$

$$E_{y\phi} = \frac{E_{y0}}{2}\left[\frac{1+Q^{-1}}{x-t} + \frac{1-Q^{-1}}{x+t}\right], \quad Q = \frac{E_{y0}}{H_{z0}}. \tag{5}$$

Self-similar solutions for fields in vacuum were also found in (2), but there the substitution is different,

$$E \sim \frac{1}{\sqrt{r}} f(\eta).$$

The substitution

Fig. 1

Figure 1: Fig. 1

$$E, H \sim \frac{1}{r} f(\eta)$$

was obtained in (3), but for another system of equations, different from (2).

Fig. 1. Oscillations of a plasma column with a longitudinal electron current and with an azimuthal initial field H_ϕ, $\rho = n_i - n_e$

Let us return to the self-similar solutions with plasma. Let the plasma be nonrelativistic. Then at $\eta = 0$ ($t = 0$) in the cylindrical case the plasma is quasineutral, and the currents are small, i.e., the fields are close to the self-similar solutions in vacuum (3)–(5). As η increases, plasma effects grow.

Figure 1 shows oscillations of a cylindrical plasma column with a longitudinal current in the field H_ϕ. The units of measurement in equations (1) (for $r = 1$ cm) are as follows: for the fields E, H, Mc^2/e; for the density, $Mc^2/4\pi e^2$; for the velocity, the speed of light c. The initial conditions of Fig. 1 are: $v_{z0} = 0.02$, $n_0 = 1$, $H_{\phi 0} = 10^{-3}$, i.e., for $r = 1$ cm, $n = 5 \cdot 10^{14}$ cm^{-3}, $H_\phi = 3$ kOe. The Alfvén velocity

$$v_A = \frac{H}{\sqrt{n}} \ll 1,$$

so the motion is nonrelativistic. The current nev_z creates the variable field H_ϕ. The field H_ϕ generates a vortex electric field E_z, which sets the electrons in motion and gives rise to ($v_z' = -\lambda E_z n/n_0$; $E_z' = nv_z$) Langmuir oscillations of the inhomogeneous plasma:

$$v_z = v_{z0} \cos \sqrt{\lambda n_0} \eta = v_{z0} \cos \omega_0 t;$$

$$\omega_0^2(r) = \frac{4\pi e^2 n_0(r)}{m};$$

$$\lambda = \frac{M}{m},$$

their period

$$T = \frac{2\pi}{\sqrt{\lambda n_0}} = 0.15$$

Fig. 2

Figure 2: Fig. 2

Fig. 3. Self-similar wave in a plasma column in a radial electric field (relativistic case)

Figure 3: Fig. 3. Self-similar wave in a plasma column in a radial electric field (relativistic case)

agrees with the results of Fig. 1.

Fig. 2. Spreading of a plasma column in a longitudinal magnetic field H_z

Figure 2 shows the evolution of the column in an initial longitudinal field ($H_{z0} = 0.2$, $u_{z0} = 0.02$, $n_0 = 0.7$; $v_A \lesssim 1$). The field H_z generates, according to the self-similar solution in vacuum (3), the component E_ϕ, and this, in turn, generates the electron current v_z. For $\eta > 0.3$, charge-separation effects are substantial, and a maximum also arises

of electron density, moving outward. Oscillations still occur, but they are of a different nature–not Langmuir oscillations ($E'_\varphi = H_z + nv_\varphi$, but $H_z > nv_\varphi$, i.e., for $\eta < 1$ the plasma is still quasineutral). The particles are accelerated to relativistic energies.

Fig. 3. Self-similar wave in a plasma column in a radial electric field (relativistic case)

Expansion of a plasma column in an electric field E_r (Fig. 3). At $t = 0$, $E_r = 0.04$, $v_{r0} = 0.005$. As can be seen, the electrons also reach relativistic velocities. For $\eta < 1$ and small v, the oscillations become Langmuir oscillations $(n = n_0,\ E'_r = nv_r,\ v'_r - \lambda \frac{n}{n_0} E_r)$.

Fig. 4. Stationary two-dimensional flow of plasma with a magnetic field. $H_{z0} = 2 \cdot 10^{-4}$; $n_0 = 0.1$; $u_{z0} = v_{z0} = 0.001$

Unlike in Figs. 1–2, along the abscissa axis there is plotted not the time η, but the distance r at a fixed instant t_0. With distance from the axis, the wavelength of the self-similar wave increases; its amplitude does not change for the velocity, and decreases for the density.

Stationary two-dimensional flow of a plasma jet with a magnetic field. The following problem was also solved: in the plane $z = 0$ there are prescribed

Fig. 4. Stationary two-dimensional flow of plasma with a magnetic field. $H_{z0} = 2 \cdot 10^{-4}$; $n_0 = 0.1$; $u_{z0} = v_{z0} = 0.001$

Figure 4: Fig. 4. Stationary two-dimensional flow of plasma with a magnetic field. $H_{z0} = 2 \cdot 10^{-4}$; $n_0 = 0.1$; $u_{z0} = v_{z0} = 0.001$

electric and (or) magnetic fields decreasing with distance from the axis as $1/r$, as well as the velocity vector of a plasma jet flowing through this plane. At $z = 0$ the density is $n_\varphi = n_0/r^2$. It is required to find a stationary ($\partial/\partial t \equiv 0$) flow in the half-space $z > 0$ satisfying these boundary conditions. In substitution (1) we replace t by z and obtain a system of self-similar equations, which we solve numerically (Fig. 4). The substitution $\eta = z/r$ makes it possible to find stationary self-similar solutions also in vacuum:

$$\begin{gathered} H_z = H_{z0}\sqrt{1+\eta^2}; \qquad H_r = H_{r0} - \eta H_{z0}\sqrt{1+\eta^2}; \\ H_x = (H_{x0} - \eta H_{z0})/(1+\eta^2); \qquad H_z = (H_{z0} + \eta H_{x0})/(1+\eta^2). \end{gathered} \tag{6}$$

At small angles $\eta \ll 1$, it is precisely the "vacuum" self-similar solutions (6) that are realized. H_z decreases, $-H_r$ increases—the lines of force bend toward the axis. The plasma also moves toward the axis ($u_r < 0,\ v_r < 0$). At large angles, space charge and the fields E_r, E_z and H_φ appear; the plasma changes the structure of the vacuum field.

Equations (2) are no longer applicable for $\eta > c/v_{\tau,e}$. In an analogous way, the region of applicability of the self-similar solutions for a plasma with hot electrons ($n_e \sim e^\varphi$, see (1)) is restricted by the conditions $\sqrt{T_e/T_i} > \eta > \sqrt{m/M}$; these inequalities make it possible to find the integral of the total number of particles and to determine the constant of the self-similar motion.

Thermal motion can be taken into account by a self-similar kinetic equation. For simplicity, we shall carry out all calculations for the planar case. We seek the electron distribution functions $\tilde{f}_e$ and ion distribution functions $\tilde{f}_i$ in the form $\tilde{f}_{e,i} = \dfrac{1}{x^2} f_{e,i}$.

Maxwell' s equations can be integrated with respect to the variable η:

$$E_x = E_{x0} + \iint (f_i - f_e) v_{xv}^2\, dv\, d\eta, \qquad E_y = \frac{1}{1-\eta^2}\left\{E_{y0} + \eta H_{z0} - \iint (f_i - f_e) v_{yv}^2\, dv\, d\eta\right\},$$

$$H_z = \frac{1}{1-\eta^2}\left\{H_{z0} + \eta E_{y0} - \eta \iint (f_i - f_e) v_{yv}^2\, dv\, d\eta\right\}.$$

Substituting these expressions into the kinetic equation

$$\frac{\partial \tilde{f}}{\partial t} + \frac{\mathbf{p}}{m\gamma}\frac{\partial f}{\partial x} + \frac{e}{m}\left(\mathbf{E} + \frac{1}{\gamma c}[\mathbf{pH}]\right)\frac{\partial \tilde{f}}{\partial \mathbf{p}} = 0; \qquad \gamma^2 = \left[\frac{p^2}{p^2+1}\right]^{-1},$$

we obtain a system of two equations for f_i and f_e with independent variables η and $\mathbf{p}$.

Applying Maxwell' s equations in integral form, we are convinced that in problems with cylindrical symmetry charges are present on the axis. In a two-dimensional flow of a cold plasma (Fig. 4), these are magnetic charges (for example, permanent magnets). In a nonstationary plasma flow with hot electrons ((1), $n_e = e^{\varphi}$), an electric charge $\varkappa T_e/e$ is present on the axis; its field prevents the electrons from escaping.

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Note: Figure translations are in progress. See original paper for figures.