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Abstract

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As is well known, a plasma pinch, i.e., a powerful gas discharge compressed by its own magnetic field, is unstable—perturbations of the necking and kink types rapidly develop on the plasma column, and it "falls apart" ([1]). This instability is well described by the theory within the framework of the magnetohydrodynamic approximation. But despite the fact that the theory of pinch stability has been developed in sufficient detail, a very interesting case of low density escaped its attention, when a substantial change occurs in the physics of the development of oscillations in the plasma.

The point is that, as the density n_0 decreases, the ratio of the directed (current) velocity of the electrons v_0 to the sound speed $c_s = \sqrt{\gamma T_0/m_i}$, which in a pinch without a longitudinal magnetic field is of the order of the Alfvén speed $c_A = B_0/\sqrt{4\pi n_0 m_i}$ (T_0 is the temperature, m_i is the ion mass, B_0 is the azimuthal magnetic field), increases. And if the linear proton number $\Pi_p = Ne^2/m_i c^2$ (for a discharge in hydrogen) becomes less than unity, i.e., if the number of particles per linear centimeter N becomes less than 10^{16}, then v_0 exceeds c_s. In this case, to describe the plasma dynamics one clearly cannot use the traditional one-fluid approximation of magnetohydrodynamics, in which the magnetic field is regarded as frozen into the ion component of the plasma. In reality, taking account of the Hall effect, the magnetic field is frozen into the electrons, and therefore for $v_0 > c_s$ the entire physics of the development of instabilities must change. We shall consider here the limiting case $\Pi_p \ll 1$, i.e., $v_0 \gg c_s \sim c_A$. Note that in this case the ions become unmagnetized: for $T_i = T_e$ their Larmor radius exceeds the radius of the column.

To describe the plasma dynamics we shall use the equations of ideal two-fluid magnetic hydrodynamics:

$$m_i n \frac{d\mathbf{v}_i}{dt} + \nabla p = \frac{1}{c}[\mathbf{j}\mathbf{B}] = \frac{1}{4\pi}[\mathrm{rot}\,\mathbf{B}, \mathbf{B}]; \tag{1}$$

$$\nabla p_e = -en\mathbf{E} - \frac{en}{c}[\mathbf{v}_e\mathbf{B}]; \tag{2}$$

$$\partial \mathbf{B}/\partial t = -c \operatorname{rot} \mathbf{E}, \qquad \operatorname{div} \mathbf{B} = 0; \tag{3}$$

$$\partial n/\partial t = -\operatorname{div} n\mathbf{v}_e = -\operatorname{div} n\mathbf{v}_i. \tag{4}$$

Here n is the density (the number of electrons or ions per unit volume), $\mathbf{v}_i$ is the ion velocity, $\mathbf{v}_e$ the electron velocity; $\mathbf{E}$ is the electric field, $\mathbf{B}$ the magnetic field; $p_e = nT_e$ is the electron pressure, and $p = n(T_e + T_i)$ is the total pressure; the remaining notation is conventional. As is seen from (4), we assume quasineutrality of the plasma. In equation (2) we have neglected the inertial term for the electrons. As for T_e and T_i, we shall not introduce additional equations for them, assuming for simplicity that in the oscillations the pressure changes adiabatically, $pn^{-\gamma} = \text{const}$. In addition, let us assume that in the equilibrium state the temperature is constant over the cross section and is equal to T_0.

In equilibrium all quantities depend only on the coordinate r of the cylindrical coordinate system r, ϑ, z. In this case, according to (1), in the absence of pro-

of the longitudinal magnetic field, the pressure p_0, the current density $j_0 = -en_0v_0$, and the azimuthal magnetic field B_0 are related to one another by

$$\frac{dp_0}{dr} = -\frac{1}{c} j_0 B_0 = \frac{en_0}{c} v_0 B_0 = -\frac{B_0}{4\pi r}\frac{d}{dr}(rB_0). \tag{5}$$

Let us now consider small oscillations of the column, in which the deviations from the equilibrium quantities (we shall denote them by a prime) vary as $\exp(-i\omega t + im\vartheta + ik_z z)$. For $v_0 \gg c_s$ the ion current in small oscillations must be much smaller than the electron current, since their orders are determined, respectively, by the quantities c_s and v_0. Therefore, in the limiting case $P_p \ll 1$ one may put

$$\mathbf{j} = -en\mathbf{v}_e. \tag{6}$$

Linearizing equations (1)–(4), (6), it is not difficult to obtain the equations for small oscillations. Let us first consider perturbations of the pinch type $m = 0$. In such perturbations $B'_r = B'_z = 0$, and for the azimuthal component of the magnetic-field perturbation, after substituting $\mathbf{E}'$ from (2) into the first equation (3), we obtain

$$(\omega - \omega_\Gamma)B'_\vartheta = -k_{zv}0B_0 n'/n_0, \tag{7}$$

where we have introduced the notation

$$\omega_\Gamma = -\frac{ckB_0}{4\pi e n_0^2}\frac{1}{r^2}\frac{d}{dr}(r^2 n_0). \tag{8}$$

In order of magnitude $\omega_\Gamma \sim v_0 k_z$, i.e. ω_Γ is a large quantity. From equation (7) it is evident that for stationary ions, $n' = 0$, the frequency $\omega = \omega_\Gamma$, i.e. ω_Γ has the meaning of the frequency of inertia-free oscillations of the helicon type. For $n_0 = \text{const}$ and $v_0 = \text{const}$, as is easy to verify, the frequency ω_Γ is simply equal to $k_{zv}0$, i.e. it corresponds to the transport of field perturbations with the electrons, but in the more general case these are indeed certain oscillations of the electron gas.

Taking the divergence of the linearized equation (1) and eliminating v_i with the aid of the continuity equation (4), we obtain the second equation for n', B'_ϑ:

$$(m_i\omega^2 + \gamma T_0 \Delta)n' = -\left(\Delta + \frac{2}{r}\frac{d}{dr}\right)\frac{B_0 B'_\vartheta}{4\pi}, \tag{9}$$

where

$$\Delta = \frac{1}{r}\frac{d}{dr}r\frac{d}{dr} - k_z^2.$$

For short-wavelength oscillations, when k_z and k_r are much larger than the inverse radius of the pinch, one may use the quasiclassical approximation, i.e. replace Δ in (9) by $-k^2 = -k_r^2 - k_z^2$:

$$(\omega^2 - c_s^2 k^2)n' = c_A^2 k^2 n_0 \frac{B'_\vartheta}{B_0}, \tag{10}$$

where $c_s^2 = \gamma T_0/m_i,\ c_A^2 = B_0^2/4\pi n_0 m_i$.

From the compatibility condition for (7), (10) we obtain the dispersion equation:

$$\omega^2 = c_s^2 k^2 - c_A^2 k^2 k_{zv} 0/(\omega - \omega_\Gamma). \tag{11}$$

Here ω_Γ is a large quantity, provided only that we do not come close to the special point where $\frac{d}{dr}(r^2 n_0) = 0$. Therefore in the second term of (11) one may neglect ω in the denominator, and its sign will be determined by the sign of v_0/ω_Γ. Since for $j_0 > 0$, $v_0 = -j_0/en_0 < 0$, the second term is positive in the inner region, $\frac{d}{dr}(r^2 n_0) > 0$, and negative in the outer region. It follows that the outer region of the pinch, where, incidentally, c_A increases with r, will be unstable with respect to pinches, at least not very far from the point where $n_0 \sim r^{-2}$. Near the special point in (11) one may neglect ω_Γ and retain only the second term, $\omega^3 = -k_{zv}0 c_A^2 k^2$. In this case one of the three roots has a positive imaginary part, corres-

corresponding to the growth increment of small oscillations $\gamma \sim c_A k_z (k_z v_0/k c_A)^{1/3}$. It can be interpreted as the result of a resonance (intersection of branches) between the Alfvén wave and a helicon with small frequency. With the aid of equation (11) one can analyze the stability of the periphery of the pinch, where the density can be approximated by a power function $n \sim r^{-\alpha}$. It turns out that, in order to stabilize the instability, i.e., for the first term in (11) to exceed the second, large α are required, i.e., a very steep decrease of density at the periphery.

Let us pass to perturbations with $m \neq 0$. In this case the condition $\operatorname{div} \mathbf{B} = 0$ and the equation for the frozen-in field in the electrons, $\partial \mathbf{B}/\partial t = \operatorname{rot}[\mathbf{v}_e, \mathbf{B}]$, which is a consequence of (2), (3) for $T_0 = \text{const}$, allow one, after eliminating B'_z, to obtain the following equations for $\psi = rB'_r$ and B'_ϑ:

$$B'_\vartheta = \frac{im}{\beta}\left(\frac{d\psi}{dr} + \frac{\omega - k_z v_0}{c_\Gamma}\frac{k^2 r^2}{m^2}\psi\right), \tag{12}$$

$$\Delta^*\psi + i\frac{k_z}{mc_\Gamma}\frac{dv_0}{dr}\psi + \frac{(\omega - k_z v_0)k^2 r^2}{c_\Gamma \beta m^2}\left(\frac{1}{r} - \frac{d\ln c_\Gamma}{dr} + \frac{\omega - k_z v_0}{c_\Gamma} + \frac{2m^2}{r\beta}\right)\psi = -\frac{i}{m}\frac{4\pi e v_0}{c}n', \tag{13}$$

where

$$\Delta^*\psi = \frac{1}{r}\frac{d}{dr}\left(\frac{r}{\beta}\frac{d\psi}{dr}\right) - \frac{\psi}{r^2}, \qquad \beta = m^2 + k_z^2 r^2, \qquad c_\Gamma = ck_{zB}0/4\pi e n_0.$$

In order not to complicate the discussion, in what follows we shall assume $v_0 =$ const and omit the second term on the left-hand side of (13). As for $m = 0$, equation (13) with $n' = 0$ describes electron oscillations of the helicon type. Naturally, in this case as well the largest increment should be expected when the helicon frequency is small, i.e., the homogeneous equation (13) has eigen-solutions with $\omega \approx 0$. Since the free term of this equation decreases rapidly with m, as m^{-2}, for large m there are certainly no eigenfunctions with small ω; therefore it is natural to restrict oneself to the case $m = 1$. Moreover, since the free term is proportional to the density and is small at the periphery, it is sufficient to restrict the discussion to the near-axis region, where $k_{zv}0/c_\Gamma \sim -2/r$, $d\ln c_\Gamma/dr \sim 1/r$, so that the homogeneous equation (13) for $\omega = 0$ takes the form

$$\Delta^*\psi + 4\frac{\beta+1}{\beta^2}k_z^2\psi = 0. \tag{14}$$

Assuming that k_z is large, $k_{zr} \gg 1$, we again restrict ourselves to the quasiclassical approximation, which shows that the value $\omega = 0$ corresponds to a plane wave with $k_r^2 = 3k_z^2$. If we choose precisely this wave, then for the inhomogeneous equation (13), taking into account terms of order $\sim \omega$, we obtain:

$$+\frac{8\omega}{kv_0}\psi = -i\frac{4\pi ev_0}{c}r^2n'. \tag{15}$$

On the other hand, from (12), for $kr \gg 1$, it follows that

$$B'_\vartheta = \frac{2i}{r}\psi. \tag{16}$$

Further, for $k_r \sim k_z \gg m/r$, in equation (1) the largest contribution is still given by the terms with $j_{rB}0$ and $j_{zB}0$, i.e., we can again use equation (9), or, more precisely, its quasiclassical approximation (10). From equations (10), (15), (16) we find the dispersion equation

$$\omega^2 = c_s^2k^2 + c_A^2k^2\frac{v_0^2k_z^2r}{4c_\Gamma\omega}. \tag{17}$$

Hence it is seen that on the mode $m = 1$ a strong instability can also develop, with increment $\gamma \sim c_{Ak}(v_0/c_A)^{1/3}$.

Thus, we have shown that in a low-density pinch sufficiently strong instabilities must develop, corresponding to resonance between

helicons and Alfvén waves. In this case, as in the case of the ordinary diffusion pinch ([2]), kink modes ($m = 1$) develop predominantly in the inner region, while sausage modes ($m = 0$) develop at the periphery.

We have considered here a pinch without a longitudinal magnetic field, but qualitatively the results obtained can also be extended to the case $B_z \sim B_\vartheta$. As for the other limiting case of a conductor in a strong magnetic field $B_z \gg B_\vartheta$, it was considered in ([3]) with the self-magnetic field B_ϑ neglected. However, as simple estimates show, this neglect is unjustified for unstable modes with small k_z, and in fact the instability considered in ([3]), due to the interaction of helicons with transverse sound, is the Kruskal–Shafranov instability expressed in another language.

The pinch instability at low density may appear experimentally in the development of a sausage-type instability in an ordinary pinch in the absence of a longitudinal magnetic field. But in the presence of $B_z \sim B_\vartheta$, as was shown in experiments on the Zeta device ([4,5]), even before $P_p \sim 1$ is reached, a competing process of electron runaway comes into play, which apparently excites oscillations and leads to the disruption of the current.

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Note: Figure translations are in progress. See original paper for figures.