A VACUUM-LIKE STATE OF A MEDIUM

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A VACUUM-LIKE STATE OF A MEDIUM AND FRIEDMANN COSMOLOGY

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The aim of this note is to show that a vacuum-like state of a physical medium (the energy-momentum tensor $T_{kl} = -\mu g_{kl}$, $\mu = \text{const}$ ([2])) can be the initial state of any of the three Friedmann models.

1. In contrast to the "singular state" (in cosmology, in the description of gravitational collapse, etc.), a vacuum-like state is completely described within the framework of general relativity (G.R.), corresponding to one of the members of the classification scheme of Einstein tensors (or energy-momentum tensors) according to the types of their algebraic structures ([1],[2]). The importance of considering it is also connected with the fact that it is a natural alternative to the idea of the inevitability of a singularity in G.R. (for example, ([3])).

If, at high density of the medium, the pressure changes sign ([4]), then the emergence of a vacuum-like state appears very probable. It is erroneously believed (for example, ([5])) that negative pressure should lead to unlimited compression of the medium. However, this does not take into account the reverse influence on the compressing medium of the changing space-time properties. Upon reaching the state with $p = -\mu$, i.e., with the energy-momentum tensor $T_{kl} = -\mu g_{kl}$, compression must cease, since by virtue of the Einstein equations one must then have $\mu = \text{const}$.

The mechanical properties of this vacuum-like state ($T_{kl} \sim g_{kl}$) and of the preceding states ($T_{kl} \nsim g_{kl}$) are qualitatively different. Namely, the latter are characterized either by the existence at each point of the medium of a unique local comoving frame of reference, defined by the fact that in it there are no fluxes of energy and momentum, or by the absence of such a frame (a free electromagnetic field). In contrast, for $T_{kl} \sim g_{kl}$ any frame of reference is comoving, for the tensor g_{kl} is diagonal in any orthonormal frame. Therefore the concept of velocity relative to a medium with $T_{kl} \sim g_{kl}$ has no meaning ([2]) –above all in this respect such a medium is similar to a vacuum. In the present work it is assumed that this qualitatively distinguished state of a medium is a special phase, in principle capable of exchanging energy, momentum, baryon charge, etc., with other phases of the medium.

The exchange of energy and momentum is easily expressed in the form of a (local) conservation law under the assumption of additivity of the total energy-momentum tensor T_{kl}. If $\tilde{T}_{kl}$ is the energy-momentum tensor of the "ordinary" ($\tilde{T}_{kl} \nsim g_{kl}$) phase of the medium, then for a two-phase medium we obtain:

$$\tilde{T}^{a}_{k;\,a} = \mu_{,k}. \tag{1}$$

Hence it is easy to show that the local increase in the density of one of the phases during a phase transition is accompanied by a corresponding change in the density of the other phase.

Let us note: the causality principle requires the non-negativity of the density μ of the vacuum-like medium; otherwise closed time-

spacelike lines. The notion of a medium with $p = -\mu$ is also introduced in (7).

The question arises of the concrete possible values of μ, above all of a possible upper bound on μ.

Let us say that interaction between elements of the medium is possible if, in the units of the proper time of each of them, the propagation time of a signal between them is finite. We shall call causally connected that part of the medium containing an element that can be in interaction with any other element of this part. Causal connectedness, in particular, determines the set of phenomena accessible to "observation" in a finite proper time of some distinguished "observer."

A world with a homogeneous and isotropic vacuum-like medium has the de Sitter metric:

$$ds^2 = A\,dt^2 - A^{-1}dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2), \qquad A = 1 - \frac{\chi}{3}\mu r^2, \tag{2}$$

where χ is Einstein' s gravitational constant.

The largest region of the medium forming a causally connected part is a sphere $r < r_0 = \sqrt{3/\chi\mu}$. Consider a part $r < r_1 \leq r_0$ of the medium. By virtue of (2) its mass is

$$m = \pi^2 r_0^3 \mu \varphi(\chi), \quad \chi \overset{\text{def}}{=} \frac{r_1}{r_0}, \quad r_0 \overset{\text{def}}{=} \left(\frac{3}{\chi\mu}\right)^{1/2}, \quad \varphi(\chi) \overset{\text{def}}{=} \frac{2}{\pi}\left(\arcsin\chi - \chi\sqrt{1-\chi^2}\right).$$

According to quantum theory, the accuracy of localization of an object of mass m cannot exceed its Compton wavelength $\lambda = \hbar/mc$. The localization region of the part of the medium under consideration is causally connected if $\hbar/mc < (3/\chi\mu)^{1/2} \arcsin\chi$. From this inequality it is easy to find that for

$$\mu \geq \mu_M \overset{\text{def}}{=} 9\pi^3 c/2\hbar\chi^2 \simeq 10^{93}\ \text{g/cm}^3$$

there is not a single part of the medium that would possess a causally connected localization region. Since the localization region of a physical object is the region of possible interactions with it, while interaction presupposes the appearance of changes in the finite proper time of at least one of the elements participating in the interaction, the absence of parts of the medium with a causally connected localization region is naturally to be understood as the cessation of all interactions in the medium. In particular, it hardly makes sense to speak of its further compression. Therefore the density $\mu = \mu_M$, evidently, should be regarded as the limiting attainable density of the medium. Let us note that it corresponds to $r_0 = (3/\chi\mu)^{1/2} \simeq 10^{-33}$ cm, i.e., to the so-called elementary length.

2. Let us consider the system of Friedmann equations

$$\ddot{a} = -\frac{\chi}{6}(\mu + 3p)a, \qquad \dot{a}^2 = \frac{\chi}{3}\mu a^2 - k \tag{3}$$

for the metric in Robertson–Walker form

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)\right]. \tag{4}$$

The parameter $a(t)$ is proportional to the distance between any two elements of the medium. Suppose that the equation of state of an extremely dense medium is $p = -\mu$. Then at the stage of contraction ($\dot{a} < 0$) the pressure p changes sign, and from the moment when $p = -\mu/3$, the acceleration $\ddot{a} > 0$. We restrict ourselves to the case in which this does not lead to a change in the sign of $\dot{a}$ before the state $p = -\mu$ is reached. In view of (3),

$$\dot{\mu} = -3(p + \mu)\dot{a}/a,$$

i.e., from the moment t_0 at which the vacuum-like state is reached, $\mu = \mu_0 =$ const. The second of equations (3) then takes the form

$$\dot{a}^2 = a^2/r_0^2 - k, \qquad r_0^2 \overset{\text{def}}{=} 3/\mu_0\chi, \tag{5}$$

and the first of equations (1) will be a consequence of (5) and the condition $p = -\mu$. Solving (5) under the assumption that at $t = t_0$, $\dot{a} < 0$, $a^2/r_0^2 - k > 0$, we obtain

$$a(t) = \frac{r_0}{2}\left(he^{-(t-t_0)/r_0} + \frac{k}{h}e^{(t-t_0)/r_0}\right); \quad k = \pm 1, 0; \quad t \geq t_0, \tag{6}$$

where $h \overset{\text{def}}{=} 2a_0/r_0 > 0$ for $k = 0$; $h = k\left(a_0/r_0 - \sqrt{a_0^2/r_0^2 - k}\right)^{-1} > 1$ for $k = \pm 1$; $a_0 = a(t_0)$.

Thus, from the instant $t = t_0$, in view of $\mu = \mu_0 = \text{const}$, the change in the state of the medium ceases, although the metric retains the form (4) with $a(t) \neq \text{const}$. This externally paradoxical result is explained by the fact that for a vacuum-like state the accompanying reference frame is not unique, and therefore for $t \geq t_0$ the parameter $a(t)$ characterizes not the motion of the medium, but only the reference frame chosen for describing the metric.

Let us show that for $t \geq t_0$ and any k, i.e., for all three Friedmann models, the metric (4) is reduced to the form (2).

We shall rely on the lemma:

If the transformations $\rho = \rho(r,t)$, $\tau = \tau(r,t)$ bring the form $\Phi = A\,dt^2 - B\,dr^2$ to the form $\Phi = \widetilde{A}\,d\tau^2 - \widetilde{A}^{-1}d\rho^2$, then

$$\widetilde{A} = \rho_r^2/B - \rho_t^2/A, \tag{7}$$

$$\tau_t = \pm\sqrt{AB}/\rho_r\left(\frac{B}{A}\frac{\rho_t^2}{\rho_r^2} - 1\right); \quad \tau_r = \frac{B}{A}\frac{\rho_t}{\rho_r}\tau_t, \tag{8}$$

where the lower indices denote taking partial derivatives with respect to the corresponding quantities.

Set $\rho \overset{\text{def}}{=} a(t)r$, $A = 1$, $B = a^2/(1 - kr^2)$. Then $\rho_r = a$, and, in view of (5), $\rho_t = \dot{a}r = -r\sqrt{a^2/r_0^2 - k}$. Substitution into (7) gives

$$\widetilde{A} = 1 - \rho^2/r_0^2,$$

whence it is clear that the form (4) is indeed reduced to the form (2), if there exists a function $\tau = \tau(r,t)$ satisfying (8). For such a function to exist it is necessary and sufficient that $\tau_{tr} = \tau_{rt}$ by virtue of (8). For $p = -\mu$ this identity is satisfied (but not in the general case of the Friedmann metric).

Thus, if the equation of state of an extremely dense medium is $p = -\mu$, then the final result of compression for all three Friedmann models ($k = \pm 1, 0$) is one and the same–the de Sitter world with a vacuum-like physical medium. Obviously, the converse is also true: a world with a vacuum-like medium may be the initial state of any of the three Friedmann worlds.

3. From what has been said it follows that the initial vacuum-like state of the medium does not determine the type of the Friedmann model. This is natural, since the corresponding metric (2) is determined by a single parameter r_0. Therefore the type of the arising Friedmann world must depend on the causes of the transition of the formally equilibrium vacuum-like state with $p = -\mu$ to the state of expansion.

The peculiarity of the vacuum-like state is the negative value of the effective gravitating density $\mu + 3p$, as a result of which any system of free particles that does not distort the metric too strongly is brought into a state of expansion. This makes probable the fluctuation instability of the vacuum-like state with respect to a transition into "ordinary matter" ($T_{kl} \not\sim g_{kl}$): the "gravitational forces" due to the properties of the vacuum-like phase tend to scatter the particles of ordinary matter that have arisen by fluctuation. By virtue of the conservation law (1), the local density of the medium will then decrease, initiating a further phase transition to the ordinary phase.

Since the matter of the ordinary phase receives an impulse toward expansion, the instability under consideration must manifest itself macroscopically in the instability of the de Sitter world with a vacuum-like medium with respect to a transition to the nonstationary state of an expanding Friedmann model. The conditions of expansion, and hence the type of the Friedmann model, are then determined by the course of the fluctuation process. Thus we arrive at a curious supposition: since the fluctuation processes of particle creation apparently must have a quantum nature and be determined by microphysical constants, the latter also determine, in the picture under consideration, the type of the emerging Friedmann world.

Since the cause of the expansion has a fluctuation character, the assumption of homogeneity and isotropy can hold only on average. The inevitability of initial inhomogeneities and non-simultaneity (in the sense of the mean Friedmann time) of the fluctuation processes gives hope of explaining the observed inhomogeneous and nonstationary structure of the metagalaxy and of discovering a connection between its characteristic parameters and microphysical constants.

It is possible that explosions in galactic nuclei represent typical features of the primordial, sharply nonstationary picture of the transition of the vacuum-like phase into the ordinary one. This implies the identification of the vacuum-like phase with the "prestellar state of matter" proposed by V. A. Ambartsumian[1].

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REFERENCES

Note: Figure translations are in progress. See original paper for figures.