ON PROBLEMS OF DISCRETE OPTIMIZATION

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Abstract

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ON PROBLEMS OF DISCRETE OPTIMIZATION

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The idea of the method, whose basic scheme will be set forth below, has recently been used for solving many problems of discrete optimization: variants of problems of production allocation ([1–6]), integer problems of linear programming ([7], [8]), problems of discrete programming with separable ([9]) and nonseparable ([10]) objective functions, as well as problems of concave programming ([11]) and problems of device reliability ([12]).

In the present note this method ψ, based on the construction of a sequence of plans, will be formulated in general form.

Let it be required to find the minimum of a function $F(x)$ defined on a finite set $P = \{p_1, p_2, \dots, p_n\}$. Thus it is required to determine an optimal element (plan) $p^* \in P$, i.e., one such that

$$F(p^*) = \min\{F(p_i) \mid p_i \in P\}.$$

The method ψ is applicable to this problem (A) if the following conditions are satisfied:

1) it is possible to find a finite extension $R = \{r_1, r_2, \dots, r_m\}$ of the set P ($P \subseteq R$) and a function $Q(x)$, defined on R, such that $Q(x) \le F(x)$ for all $x \in P$;

2) it is possible to construct an algorithm φ which, at the k-th step ($k = 1, 2, \dots$), effectively finds an element $r_{j_k} \in R$ having the property

$$Q(r_{j_k}) = \min_{x \in R_k} Q(x), \qquad k = 1, \dots, m,$$

where $R_k = R_{k-1} \setminus r_{j_{k-1}}$, and $R_1 = R$ *.

If, from the sequence $r_{j_1}, r_{j_2}, \dots$ constructed with the aid of φ, one removes the elements not belonging to the set P, then for the remaining sequence $p_{i_1}, p_{i_2}, \dots$ the following holds.

Optimality criterion. *If there exists a natural number t $(t \leq n)$ such that*

$$Q(p_{i_t}) \geq \min F(p_{i_s}) = F(p^*), \tag{1}$$

then p^* is an optimal element of problem A.

Thus, the procedure ψ for finding an optimal element of problem A is as follows. At the k-th step $(k = 1, 2, ...)$, with the aid of algorithm φ, we find an element r_{j_k} and check whether it belongs to the set P. If not, we pass to the next step of algorithm ψ. If yes $(r_{j_k} = p_{i_t})$, then we check whether condition (1) is satisfied. If yes, the process terminates and p^* is an optimal element of problem A. If not, we pass to the next step of algorithm ψ.

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* In other words, there must exist an effective method for constructing a sequence of elements of the set R in the order of nondecrease of the function $Q(x)$ (cf. ([10])).

Remarks. 1. If at the k-th step of algorithm ψ it is possible to refine the approximating function $Q(x)$, i.e., to find a function $Q_k(x)$ satisfying the condition

$$Q(x) \leq Q_{k-1}(x) \leq Q_k(x) \leq F(x)$$

for all $x \in R_k$, then the procedure can be accelerated.

2. Obviously, if there is no number t satisfying condition (1), then algorithm ψ turns into complete enumeration. However, as was indicated in ([2,11]), at each step one can estimate the deviation of the best of the obtained elements of the set P from the optimal one.

3. If the function $Q(x)$ coincides with $F(x)$ on the set P, then our procedure turns into a scheme close to the general scheme proposed in ([13]). In this case the optimal element of problem A is sought among the elements r of the set R satisfying the condition $Q(r) \leq F(p^*)$ (7).

If, moreover, a lower bound $\bar{F}$ of the values of the function $F(x)$ on the set P is known, then this accelerates the procedure, since before checking whether the element r_{j_k} belongs to the set P^*, we must make sure that $Q(r_j^k) \geq \bar{F}$ (otherwise one must pass to the next step of algorithm ψ) (7).

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* This check is often laborious; for example, in the case when P is the set of solutions of a system of inequalities ([6,7]).

Note: Figure translations are in progress. See original paper for figures.