# ON CELLULAR CONVECTION IN AN INFINITELY LONG HORIZONTAL LAYER OF GAS HEATED FROM BELOW

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Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

## Abstract

## Full Text

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# ON CELLULAR CONVECTION IN AN INFINITELY LONG HORIZONTAL LAYER OF GAS HEATED FROM BELOW

*(Presented by Academician G. I. Petrov on 19 V 1970)*

**1.** We consider the flow and heat transfer in an infinitely long horizontal layer of gas of height $H$, bounded by solid surfaces, in the field of an external body force producing an acceleration $g$ (Fig. 1). On the lower surface, at $y = 0$, a constant temperature $T_2$ is maintained; on the upper surface, the temperature is $T_1$, with $T_2 > T_1$. Thus the gas is heated from below, and its hydrostatic equilibrium in the field of the external force may be unstable.

**Fig. 1.** Mean heat transfer through the layer as a function of the Rayleigh number, defined with a correction for the magnitude of the adiabatic temperature gradient.
1–$\chi = 1.4$; $K = 0.04$; 2–$\chi = 1.0$; $K = 0$; 3–$\chi = 1.4$; $K = 0.2$; 4–$\chi = 1.66$; $K = 0.33$

**Fig. 2.** Distribution of the local Nusselt number on the lower wall.
1–$\widetilde{\mathrm{Ra}}_m = 1775$; 2–$\widetilde{\mathrm{Ra}}_m = 1880$; 3–$\widetilde{\mathrm{Ra}}_m = 3135$; 4–$\widetilde{\mathrm{Ra}}_m = 10450$; 5–$\widetilde{\mathrm{Ra}}_m = 52250$

The stability conditions for a gas heated from below can be obtained from the linearized equations of hydrodynamics and heat transfer. The onset of convection in liquids is determined by the Rayleigh number and occurs when $\mathrm{Ra} > \mathrm{Ra}_{\mathrm{cr}}$. Under the assumption of periodic perturbations that take the system out of equilibrium, with a linear initial temperature profile, the critical Rayleigh number $\mathrm{Ra}_{\mathrm{cr}}$ for a layer bounded by solid surfaces was calculated from the linearized Boussinesq equations in work [1] and is equal to 1708. This value of the critical Rayleigh number corresponds, in the plane case, to a perturbation wavelength $\lambda = L/H = 1.008$. Calculations of two-dimensional periodic cellular convection beyond the stability threshold at $L/H = 1$, based on the nonlinear equations in the Boussinesq approximation, were carried out in works ([2,3,15]).

The question of the onset of convection and of the features of heat transfer in gases beyond the stability threshold is connected with the analysis of the equations of a compressible gas and has been studied far less. The necessary condition for the onset of convection, under the assumption of adiabatic displacement of a particle from the state of equilibrium and neglecting viscosity and thermal conductivity, can be obtained from simple thermodynamic considerations without

reduction of the equations of motion ([4]) and has the form $K < 1$, where $K$ is the ratio of the adiabatic temperature gradient to the actual gradient.

Jeffreys ([5]) somewhat extended the range of applicability of the Boussinesq equations by taking into account, in the heat-transfer equation, the work of compressive forces. The condition for the onset of convection in this case has the form $\widetilde{\mathrm{Ra}} > \widetilde{\mathrm{Ra}}_{\mathrm{cr}}$, where

$$\widetilde{\mathrm{Ra}} = \frac{g\beta H^4}{\nu a}\left[\left(\frac{dT}{dy}\right) - \left(\frac{dT}{dy}\right)_{\mathrm{ad}}\right] = \mathrm{Ra}(1-K).$$

However, the conditions of applicability of this approximation, especially for developed motion beyond the stability threshold, have been little studied. In the general case of a compressible and viscous gas, the onset of convection is determined by the more complicated condition $\mathrm{Ra}(K)$. For the case of convection in a cell bounded by solid surfaces, with $L/H = 1$, the dependence $\mathrm{Ra}(K)$ was found in ([6]) by numerical solution of the full Navier–Stokes equations for a compressible gas.

2. Assuming the coefficients of viscosity $\mu$ and thermal conductivity $k$ to be constant, we write the initial equations in the form (notation generally accepted)

$$\rho(\partial V/\partial t + V\nabla V) = -\nabla P + \rho\vec{g} + \mu\nabla^2 V + \tfrac{1}{3}\mu\nabla(\nabla V), \tag{1}$$

$$\partial\rho/\partial t + \nabla(\rho V) = 0, \tag{2}$$

$$\rho C_v(\partial T/\partial t + V\nabla T) + P\nabla V = k\nabla^2 T + \mu\Phi, \tag{3}$$

$$P = \rho R T. \tag{4}$$

Here $\Phi$ is the dissipation function.

We shall assume the fields of velocity, temperature, and density to be two-dimensional, periodic in the direction normal to the direction of the force $g$; we consider a cell of width $L$, equal to half the period, with symmetry boundary conditions on the lateral surfaces, no-slip conditions, and prescribed constant temperatures on the solid surfaces $y = 0, H$.

It is not difficult to find the system of similarity criteria determining the flow and heat transfer, which we shall write in the form ([6])

$$\mathrm{Ra},\ K,\ \mathrm{Pr},\ C_F,\ \chi. \tag{5}$$

Here $\mathrm{Pr} = \nu/a$ is the Prandtl criterion, $\chi = c_p/c_v$ is the ratio of heat capacities, $C_F = gH/\chi RT_1$ is the criterion of weight compressibility, which determines the change in density as a function of the height $H$ and the magnitude of the acceleration of gravity $g$ in accordance with the equation of hydrostatics and the equation of state of the gas. The stability criterion $K$ is a combination of the criteria $\chi$, $C_F$, and the temperature ratio $T_2/T_1$, and can be written as:

$$K = (\chi - 1)C_F/(T_2/T_1 - 1). \tag{6}$$

In addition, in the adopted formulation of the problem, the dimensionless wavelength of the convective motion $L/H$ is also a parameter. In what follows we shall be interested in stationary solutions of problem (1)–(4), which we shall obtain by establishing them in the solution of the nonstationary problem by a finite-difference method ([7]). Excitation, beyond the stability threshold, of convective motions having a periodic structure was achieved by introducing nonsymmetric perturbations into the initial field of a motionless gas in order to single out a stationary solution with one vortex in the cell $L/H = 1$. Further investigation was carried out with variation of $L/H$ and the similarity criteria (5), starting from these initial data. In the calculations a uniform difference grid was used, with the number of nodes in the computational domain $21 \times 21$ and $31 \times 31$. The difference between the corresponding mean Nusselt numbers calculated on these grids at $\mathrm{Ra} \leq 10^5$ was no more than 3%. The residuals in the integral heat balances at $\mathrm{Ra} < 5 \cdot 10^4$ for station-

in the steady regime did not exceed 1%. In the calculations the numbers $\mathrm{Pr} = 1$ and $C_F = 0.05$* were fixed.

3. The results of calculating the mean Nusselt number $\overline{\mathrm{Nu}} = q_w H/k(T_2 - T_1)$ as a function of the number $\widetilde{\mathrm{Ra}}_m = \mathrm{Ra}_m(1 - \mathrm{K})$, i.e., with a correction for the value of the adiabatic gradient, for stationary motions beyond the stability threshold at $L/H = 1$, are presented in Fig. 1. The value of $\mathrm{Ra}_m$ was determined from quantities taken at the point in the layer midway in height. Curves 1-4 correspond to different values of $\chi$ and $T_2/T_1$ and cover the range of $\chi$ for monatomic and triatomic gases at superadiabatic

Fig. 3. Comparison of results of calculating the mean heat transfer through a layer at K = 0 with experimental data ([9,12,14])

Figure 3: Fig. 3. Comparison of results of calculating the mean heat transfer through a layer at K = 0 with experimental data ([9,12,14])

temperature gradients (K = 0 ÷ 0.3). The intersection of the curves with the horizontal line $\overline{\mathrm{Nu}} = 1$ determines the critical Rayleigh number, which in the range of similarity criteria (5) considered

**Fig. 3.** Comparison of results of calculating the mean heat transfer through a layer at K = 0 with experimental data ([9,12,14])

varies within the limits $(\widetilde{\mathrm{Ra}}_m)_{\mathrm{cr}} = 1620 \div 1750$. At K = 0 the value of $(\widetilde{\mathrm{Ra}}_m)_{\mathrm{cr}}$ is equal to 1750, which agrees with the results of experiments both with gases ([8]) and with liquids ([9]) within the scatter of these experimental data (in these experiments the value of K was not more than $10^{-2}$), but is somewhat higher than the value $\mathrm{Ra}_{\mathrm{cr}} = 1708$, which is obtained from the Boussinesq equations. With increasing distance from the stability threshold, the mean heat transfer (Fig. 1) is no longer determined only by the number $\widetilde{\mathrm{Ra}}$, but also depends on other similarity criteria characterizing the gas; in particular, the heat transfer decreases as K($\chi$) increases.

4. For small $\mathrm{Ra} - \mathrm{Ra}_{\mathrm{cr}}$, the heat flux from the heated wall at its initial section along the flow is higher, and farther downstream is lower, than in the case when convection is completely absent and the number $\mathrm{Nu}_L = 1$ (Fig. 2). The excess of the heat flux from the heated wall over its entire surface over the heat flux when convection is absent actually occurs only sufficiently far from the stability threshold and in the range of Ra numbers where the flow is already, apparently, turbulent. This "incompleteness of mixing" is apparently a general property of convective flows of liquids and gases in closed regions heated from below and from the side ([10]), at small Ra numbers (or $\mathrm{Ra} - \mathrm{Ra}_{\mathrm{cr}}$).

5. Experimental data for heat transfer through gas layers exist only for the case when the temperature gradient is much greater than the adiabatic gradient and weight compressibility is practically absent, i.e., in the case when the gas and liquid models are equivalent. However, the satisfactory agreement of experimental data with those calculated on the basis of the roll model at the corresponding Ra numbers makes it possible to assume that such a model is also valid in cases specific to a gas, in the corresponding range of $\widetilde{\mathrm{Ra}}$.

* This value of $C_F$ corresponds to not very deep layers, in which the relative change in density as a function of pressure is not more than 5%.

The roll structure, according to experiments, is observed in layers of liquid and gas enclosed between solid cylindrical disks made of a well-conducting material

Fig. 4

Figure 4: Fig. 4

(11). The most complete measurements of the mean heat transfer under such conditions were carried out by Silveston (9)* and Rossby (12). In the range $\mathrm{Ra}_m \leqslant 5 \cdot 10^3$, the results of the calculation agree well with the results of the measurements (9, 12, 14) and exceed them with further increase of the number $\mathrm{Ra}_m$; moreover, the relative error at $\mathrm{Ra}_m = 10^5$ is about 15% (Fig. 3). This is apparently connected with the disturbance of the roll structure of the flow far from the stability threshold and with the transition to a turbulent flow regime. A similar result was also obtained in (2, 3) when comparing the results of numerical solutions of the Boussinesq equations for a cell $L/H = 1$ with Rossby' s data (12).

6. The range of applicability of the roll-convection model can be somewhat broadened if one takes into account that the wavelength of convective motions beyond the stability threshold changes. The analysis method used does not make it possible to answer the question of which wavelength motions must be realized, but it does make it possible to investigate the flow structure and heat transfer in cells as a function of wavelength. This dependence, for three values of the Rayleigh number beyond the stability threshold, is shown in Fig. 4. The dashed lines indicate the values of the number $\overline{\mathrm{Nu}}$ corresponding to the experimental data. It is seen from this that, upon moving away from the stability threshold, the maximum value of the mean heat transfer shifts toward convective motions with a shorter wavelength. However, most authors (9, 12, 13) note that in experiments the wavelength beyond the stability threshold increases. Figure 4 shows that the calculated values of the mean Nusselt number in this case agree with the measured ones up to $\mathrm{Ra}_m \approx 10^4$. A more detailed study of the structure of cellular convection is contained in (16).

**Fig. 4.** Influence of the wavelength of the convective motion on heat transfer beyond the stability threshold.
1 $-\mathrm{Ra}_m = 2090$; 2 $-\mathrm{Ra}_m = 5225$; 3 $-\mathrm{Ra}_m = 10450$.

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## REFERENCES

1. A. **Pellow**, R. V. **Southwell**, *Proc. Roy. Soc.*, Ser. A, **176**, 312 (1940).

2. P. **Schneck**, G. **Veronis**, *Phys. Fluids*, **10**, No. 5, 927 (1967).

3. W. H. **Plows**, *Phys. Fluids*, **11**, No. 8, 1593 (1968).

4. K. **Schwarzschild**, *Göttingen Nachrichten*, 11 (1906).

5. H. **Jeffreys**, *Proc. Camb. Phil. Soc.*, **26**, 170 (1930).

6. V. I. **Polezhaev**, *Izv. AN SSSR, Mekh. zhidkosti i gaza*, No. 6, 97 (1968).

7. V. I. **Polezhaev**, ibid., No. 2, 103 (1967).

8. H. A. **Thompson**, H. H. **Sogin**, *J. Fluid Mech.*, **24**, Part 3, 451 (1966).

9. P. L. **Silveston**, *Forsch. Ing.-Wes.*, **24**, Nos. 1, 2 (1958).

10. V. I. **Polezhaev**, *Izv. AN SSSR, Mekh. zhidkosti i gaza*, No. 4 (1970).

11. E. L. **Koschmieder**, *Beitr. Phys. Atmos.*, **39**, 1 (1966).

12. H. T. **Rossby**, *J. Fluid Mech.*, **36**, Part 2, 309 (1969).

13. E. L. **Koschmieder**, *J. Fluid Mech.*, **35**, Part 3, 527 (1969).

14. W. **Mull**, H. **Reiher**, *Beih. Gesundh.-Ing.*, Reihe 1, No. 28 (1930).

15. L. L. **Shvartsblat**, *Izv. AN SSSR, Mekh. zhidkosti i gaza*, No. 5, 84 (1969).

16. M. P. **Vlasyuk**, V. I. **Polezhaev**, Preprint No. 37, Institute of Applied Mathematics, Academy of Sciences of the USSR (1970).


* In Silveston' s flow-visualization experiments a more complicated three-dimensional flow structure was observed, which, as indicated in (13), was connected with the influence of the poorly conducting upper surface made of org-glass, whereas the heat-transfer measurements were carried out with a metallic upper surface.

*Note: Figure translations are in progress. See original paper for figures.*