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Abstract

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THE *PP* WAVE IN ATOMIC EXPLOSIONS AND THE ATTENUATION OF LONGITUDINAL WAVES IN THE UPPER MANTLE

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The amplitudes of the PP wave are analyzed—a longitudinal wave reflected from the Earth' s surface at half the epicentral distance. The principle of analysis described in ([1]) is employed; it consists in the following. The problem of calibrating seismic sources, reliably calibrating the instruments, and allowing for the conditions of their installation can be avoided if, instead of the absolute values of the amplitudes of different phases of body waves, one measures their ratio on the same seismogram. A characteristic feature of body waves is a large scatter of amplitudes caused by a number of factors. There are grounds for assuming that the amplitude of a seismic wave, as a random quantity, is distributed lognormally. All the information that in this case can be extracted from observations is contained in the mean μ and the variance σ^2 of the logarithm of the amplitude ratio (averaging over the coordinates of sources and receivers is implied). The scatter of amplitudes leads to the fact that the wave observed in later arrivals can be identified only on part of the seismograms. μ and σ^2 can also be found in this case by using the method of ([1]). In the N cases when both waves are identified, β_i—the ratio of their amplitudes—is measured; in the M cases when one of them is not visible against the noise background, α_j —the ratio of the background amplitude to the amplitude of the identified wave —is measured.

The parameters μ and σ are determined from the maximum of the likelihood function

$$L(\mu,\sigma) = -\sum_{i=1}^{N} \frac{(\ln\beta_i-\mu)^2}{2\sigma^2} - N\ln\sigma + \sum_{j=1}^{M} \ln\left[1+\Phi\left(\frac{\ln\alpha_j-\mu}{\sqrt{2}\sigma}\right)\right] + C, \quad (1)$$

where

Fig. 1

Figure 1: Fig. 1

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt;$$

C is a normalization constant.

The mean ratio of the energy-flux densities λ^2 can be found from the formula

$$\lambda^2 = \exp 2(\mu + \sigma^2 - 2\sigma_p^2),$$

where σ_p^2 is the variance of the logarithm of the amplitude of the wave appearing in the denominator of the ratio. For the longitudinal wave of underground explosions with a frequency of about 1 Hz, we currently estimate σ_p^2 as 0.3–0.4.

The results of the analysis of seismograms of underground nuclear explosions, carried out at different test sites and recorded by short-period instruments at seismic stations in the USSR, are used. All the data are independent in the sense of ([1]). The predominant frequency of oscillations in the longitudinal waves is always about 1 Hz.

Figure 1 presents the results of measuring A_{PP}/A_P. Figure 2 shows function (1) for $\Delta = 28$–37°. The estimates of μ, σ, and λ in this case are -1.35; 1.1; 0.45. For the range of Δ from 85 to 101°, obtained

estimates are $\mu = -1.5$, $\sigma = 1.0$, $\lambda = 0.27$. In the remaining ranges, where there are few values of β, the estimates of μ and especially of σ are extremely unreliable. In the range Δ from 85 to 101°, the values of A_{PP}/A_P clearly depend on epicentral distance; therefore the estimates of μ and σ here are of a formal character, and the value of σ is overestimated because of this dependence. In analogous calculations for the PcP wave reflected from the core, $\sigma \approx 1.3$ ([1]) was obtained; in this case σ_{PcP}^2 proved to be approximately three times greater than σ_P^2. This result made it possible to suggest that in the lower mantle there is a zone that is strongly heterogeneous horizontally. The scatter of amplitudes may be regarded as an indicator of the degree of horizontal (lateral) heterogeneity of the medium, including in the concept of heterogeneity also the unevenness of the reflecting surface, if a reflected wave is involved. Comparison of data on the scatter of the amplitudes of PP and PcP waves makes it possible to suggest that the hypothetical zone in the lower mantle is comparable in degree of horizontal heterogeneity with the crust and the upper mantle.

Fig. 1

Fig. 2

Fig. 2

Figure 2: Fig. 2

Analysis of the amplitudes of the PP wave makes it possible to approach an estimate of the absorption of longitudinal waves in the upper mantle. The difficulties of this problem are connected primarily with the fact that changes in the amplitude of the refracted wave with distance depend not only on absorption but also on the geometrical factor. To take the geometrical factor into account, one requires reliable knowledge of the second derivative of the hodograph or of the derivative of velocity with depth, but these quantities are measured with low accuracy. The problem of estimating the geometrical factor can be avoided by comparing the spectra of the wave at different epicentral distances or the spectra of different waves. This method, popular in recent years, has made it possible to estimate relative changes in attenuation parameters for various rays, but has proved less effective in attempts to obtain absolute estimates. Below another approach to this problem is considered.

Let us assume that the amplitude curve $A_P(\Delta)$ for a given frequency f is known at two epicentral distances: Δ_1 and $\Delta_2 = 2\Delta_1$, and that the value $A_{PP}(\Delta_2)/A_P(\Delta_2)$ is known. Without taking absorption into account, the energy of a disturbance from a symmetric source at the surface, referred to a unit area of the wave front with emergence angle i, is equal to ([2])

$$E = \frac{c}{\sin\Delta\ \operatorname{tg} i}\left|\frac{di}{d\Delta}\right|,$$

where c is a constant. Taking into account absorption and reflection, and considering the wave to be sinu-

sinusoidal, we obtain

$$A_{PP}^2(\Delta_2) = \frac{r^2 A_P^2(\Delta_1)\sin\Delta_1 \exp[-2\pi f T(\Delta_1)/\overline{Q}_\alpha(\Delta_1)]}{2\sin\Delta_2}, \tag{2}$$

where r is the amplitude reflection coefficient, $T(\Delta_1)$ is the travel time of the longitudinal wave for a ray emerging at epicentral distance Δ_1; $\overline{Q}_\alpha(\Delta_1)$ is the effective quality factor of the medium for this ray. In reality A_P and A_{PP} have scatter, which was discussed above. Taking this scatter into account, formula (2) may be written as follows:

$$\langle A_{PP}^2(\Delta_2)\rangle = \frac{r^2 \langle A_P^2(\Delta_1)\rangle \sin\Delta_1 \exp[-2\pi f T(\Delta_1)/\overline{Q}_\alpha(\Delta_1)]}{2\sin\Delta_2}, \tag{3}$$

where the brackets denote averaging over the coordinates of sources and receivers. For a logarithmically normal distribution of amplitudes, the relation

Fig. 3

Figure 3: Fig. 3

$$\langle A^2 \rangle = \exp 2(\mu + \sigma^2), \tag{4}$$

is valid, where $\mu = \langle \ln A \rangle$, $\sigma^2 = \langle (A - \langle \ln A \rangle)^2 \rangle$. Using (3) and (4), we write the relation:

$$H(\Delta_1) = \pi f T(\Delta_1)/\overline{Q}_\alpha(\Delta_1) = \ln r + \mu_P(\Delta_1) + \sigma_P^2(\Delta_1) - \mu_P(\Delta_2) - \sigma_P^2(\Delta_2) - \mu_{PP}(\Delta_2) - \sigma_{PP}^2(\Delta_2) + \mu_P(\Delta_2) + \sigma_P^2(\Delta_2) + \ln\sqrt{\sin\Delta_1} - \ln\sqrt{2\sin\Delta_2}. \tag{5}$$

Naturally one may take $\sigma_P^2(\Delta_1) = \sigma_P^2(\Delta_2)$; in addition, we assume that $\sigma_{PP}^2(\Delta_2) = \sigma_P^2(\Delta_2)$. If the second assumption is erroneous and in reality $\sigma_{PP}^2 > \sigma_P^2$, then as a result the value of $\overline{Q}_\alpha$ will be underestimated. The final expression for $H(\Delta_1)$ takes the form:

$$H(\Delta_1) = \pi f T(\Delta_1)/\overline{Q}_\alpha(\Delta_1) = \ln r + \langle \ln A_P(\Delta_1) \rangle - \langle \ln A_P(\Delta_2) \rangle - \langle \ln[A_{PP}(\Delta_2)/A_P(\Delta_2)] \rangle + \ln\sqrt{\sin\Delta_1} - \ln\sqrt{2\sin\Delta_2}. \tag{6}$$

Fig. 3

We shall use empirical data on amplitudes of the P wave from nuclear explosions, recorded by the WWSSN network and by stations in Canada ([3]). These data, like ours, refer to a frequency of 1 Hz; the measurement procedure adopted in ([3]) also does not differ from ours. As Δ_1 we take distances 42.5–50°, and as Δ_2, distances from 85 to 100°. According to the data of ([3]), $\langle \ln A_P \rangle$ in the interval 42.5–50° may be estimated as 10.2 ± 0.12. The amplitude curve in the interval 85–100° is approximated by an inclined straight line (Fig. 3a). The values $\langle \ln A_{PP}(\Delta_2)/A_P(\Delta_2) \rangle$ in the same interval are approximated by a straight line having the same magnitude but an opposite sign of slope (Fig. 3b). The procedure of this approximation is based on the maximum-likelihood method. As a result, the quantity $\langle \ln A_P(\Delta_2) \rangle + \langle \ln A_{PP}(\Delta_2)/A_P(\Delta_2) \rangle$ is estimated as 7.2 ± 0.3 and is assigned to $\Delta = 93°$. The value r (all reflections occurred in the ocean) is taken equal to 0.9. If this figure is too high, then the estimate of $\overline{Q}_\alpha$ will be

underestimated. As a result, the value $H(46.5°)$ from formula (6) is estimated as 2.4 ± 0.3. Let us assume that the admissible values of $H(46.5°)$ lie in the interval 2.4 ± 0.6. The predominant frequency of oscillations in the P wave is 1 Hz, and in the PP wave, 0.8 Hz; we take $f = 0.9$ Hz. Then $480 < Q_\alpha(46.5°) < 800$. The corresponding ray penetrates to a depth of about 1100 km.

Let us compare the result obtained with other estimates of Q for the upper mantle. We represent Q_α in the form ηQ_β, where Q_β is the distribution of the quality factor for long-period ($f \approx 0.2$–0.003 Hz) transverse waves, according to Anderson et al. ([4]). We have found that $1.5 < \eta < 2.5$. Values obtained in other works are given in Table 1.

Table 1

Frequency, Hz	η	Literature source
~ 1	~ 1	([5])
~ 1	~ 1–1.30	([6])
~ 0.03	~ 1	([7])
~ 0.03	~ 1	([8])
0.16	≥ 10	([9])

Thus, our estimate exceeds most of the others. It is known that if the mechanism of absorption of seismic energy is such that losses under pure compression are absent, then one should have $\eta \approx 2$–2.5 ([4]). Our result does not contradict this assumption if it is additionally assumed that Q is practically independent of frequency in the range from 1 to 0.003 Hz. In conclusion, we emphasize the necessity of checking the result obtained on more numerous data.

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Note: Figure translations are in progress. See original paper for figures.