ON THE THEORY OF THE GEYSER PROCESS

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ON THE THEORY OF THE GEYSER PROCESS

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In works ([1–5]), various views have been expressed on the mechanism of action of a geyser. However, there is no physical theory that would make it possible to characterize the geyser regime quantitatively. An attempt to construct such a theory is undertaken in the present work.

The following ideas about the mechanism of the phenomenon are taken as the basis of the discussion. The eruption of a geyser occurs as a result of the boiling of water accumulating in an underground reservoir (chamber). The working body of a geyser is groundwater entering the reservoir, and the source of heating is underground heat. The heating mechanism may be different—from the walls of the chamber, through mixing with high-temperature waters or superheated condensing steam. The eruption of a geyser has the character of an explosion, i.e., it occurs as a result of a rapid volumetric release of energy. The source of the explosion is the energy of superheating of the water (accumulated heat), which is connected with the dependence of the boiling temperature on pressure and is equal to

$$W = c(T_{\mathrm{k}2} - T_{\mathrm{k}1}),$$

where $T_{\mathrm{k}1}$ is the boiling temperature of water at atmospheric pressure P_1; $T_{\mathrm{k}2}$ is the boiling temperature of water at the pressure P_2 corresponding to the depth of occurrence of the chamber h; $P_2 = P_1 + \rho g h$; ρ and c are, respectively, the density and specific heat capacity of water; g is the acceleration due to gravity. The greater the value of $(\rho g h / P_1)$, the greater the superheating energy, the more strongly the phenomenon of the geyser explosion is manifested. The geyser process has a clearly expressed induction period, during which the water entering the chamber is heated. At the temperature in the chamber $T_{\mathrm{k}2}$, a progressive self-accelerating drop of pressure in the chamber to the value P_1 occurs, associated with the formation of bubbles during boiling and the unloading of the channel. This leads to intense steam formation and eruption.

Fig. 1. Schematic of a geyser system.

Figure 1: Fig. 1. Schematic of a geyser system.

Fig. 1. Schematic of a geyser system. a —collector of cold groundwater; b —aquifer; v —porous-fractured collector of high-temperature waters; g —inflow of cold waters; d —inflow of high-temperature waters.

geyser. The process ends when all the energy of superheating has been expended on vaporization and on the motion of the two-phase mixture.

Manifestations of the geyser process in nature may be highly varied because of differences in the geometry and volumes of geyser reservoirs, the rates and paths of cold-water supply, and the heating mechanism. Without striving for quantitative universality of the results, but also without reducing the generality of the approach, let us consider the following simple geyser model (Fig. 1). We shall assume that:

a) Heating of the "cold" water in the chamber is accomplished mainly by mixing with a hot heat carrier, while heating from the chamber walls may be neglected. Mixing occurs rapidly, so that the temperature is the same at all points of the chamber.

b) Cold water from the aquifer is supplied directly into the chamber, and not through the geyser conduit.

c) The mass flow rate of the hot heat carrier G_r, with equivalent temperature T_r *, does not depend on the pressure in the chamber P, while that of the cold water T_x is described by the relation $G_x = a(P_* - P)$, where P_* is the pressure in the aquifer and a is the hydraulic permeability.

d) The chamber volume V_1 is greater than the volume of the conduit V_k; the thermal and hydraulic state of the water in the conduit does not affect the heat balance of the chamber; the pressure drop inside the chamber is insignificant.

The aim of the present work is a theoretical analysis of the induction pattern of the geyser process (from the end of an eruption to the beginning of the next one). Taking into account that the duration of the eruption itself is usually much shorter than the induction time t_{ind}, the value t_{ind} may be taken, with satisfactory accuracy, as the period of the geyser regime.

Let us consider three stages of the process.

1st stage —filling of the chamber. After an eruption, part of the chamber is filled with residual water having temperature T_{k1}. As a result of the inflow of "cold" and hot water (or condensing steam), the volume of water in the chamber increases and its temperature changes. The pressure of the water in the chamber and the flow rate of "cold" water are constant and equal, respectively, to P_1

and G_{x1}. The process is described by the system of equations of mass and heat balance:

$$\rho(dV/dt) = G_1; \tag{1}$$

$$\rho V(dT/dt) = G_r(T_r - T) + G_{x1}(T_x - T) \tag{2}$$

with initial conditions $t = 0$, $V = V_0$, $T = T_{k1}$. Notation: V is the volume of water (current value); V_0 is the residual volume of water in the chamber after eruption; T is the water temperature; t is time; $G_1 = G_r + G_{x1}$ is the total liquid flow rate.

The solution of equations (1) and (2) has the form

$$V = V_0 + (G_1/\rho)t; \tag{3}$$

$$T = T_{p1} + (T_{k1} - T_{p1})\left[1 + (G_1/\rho V_0)t\right]^{-1}, \tag{4}$$

where $T_{p1} = (G_{x1}T_x + G_rT_r)/G_1$ is the equilibrium temperature of the two mixing streams. In what follows we shall consider the case $T_{p1} < T_{k1}$, which is of greatest interest for describing the geyser regime.

It is characteristic that the temperature of the water in the chamber decreases during its filling. The filling time of the chamber and the water temperature at the end of the first stage, as follows from (3), (4), are

$$t_1 = (\rho/G_1)(V_1 - V_0); \tag{5}$$

$$T_1 = T_{p1} + (V_0/V_1)(T_{k1} - T_{p1}). \tag{6}$$

* In the case of condensing steam, $T_r \approx T_p + (L/c)$, where T_p is the steam temperature and L is the heat of vaporization.

2nd stage–filling of the channel. The water pressure in the chamber increases, the flow rate of "cold" water G_x decreases, the volume of water in the chamber does not change and is equal to V_1. The change in P is determined by the change in the water level in the channel. It is not difficult to show that

$$(dp/dt) = (g/s)[G_1 - a(P - P_1)] \tag{7}$$

for $t = t_1$, $P = P_1$, where $s = V_k/h$ is the effective cross section of the channel. From (7) it follows that

$$P = P_1 + (G_1/a)\{1 - \exp[-(ag/s)(t - t_1)]\}. \tag{8}$$

The expressions for G_x and G have the form

$$G_x = G_{x1} - G_1\{1 - \exp[-(ag/s)(t - t_1)]\},$$

$$G = G_1 \exp[-(ag/s)(t - t_1)]. \tag{9}$$

The end of the 2nd stage corresponds to the emergence of geyser water at the surface (we do not consider the influence of the volume of the vent on the laws of pressure variation in time), i.e., at $t = t_2$, $P = P_2 = P_1 + \rho g h$, whence

$$t_2 = t_1 - (s/ag)\ln[1 - (a\rho g h/G_1)]. \tag{10}$$

The heat-balance equation for the water in the chamber, taking (9) into account, can be written in the form

$$(\rho V_1/G_1)(dT/dt) = (T_{p1} - T_x) - (T - T_x)\exp[-(ag/s)(t - t_1)]. \tag{11}$$

The initial conditions are $t = t_1$, $T = T_1$.

The solution of equation (11) is expressed through exponential-integral functions

$$T = T_x + (T_{p1} - T_x)\{\theta_1 e^{-B} - B[\mathrm{Ei}(-Be^{-\tau}) - \mathrm{Ei}(-B)]\}\exp(Be^{-\tau}), \tag{12}$$

where

$$\mathrm{Ei}(x) = \int_{-\infty}^{x} \frac{e^x}{x}\,dx,$$

$$\tau = (1 - \beta)(G_1/\rho V_k)(t - t_1), \qquad \theta_1 = (T_1 - T_x)/(T_{p1} - T_x),$$

$$B = (V_k/V_1)(1 - \beta)^{-1}, \qquad \beta = (G_2/G_1) = 1 - (a\rho g h/G_1). \tag{13}$$

The temperature at the end of the 2nd stage (at $\tau = \tau_2 = -\ln\beta$) is equal to

$$T_2 = T_x + (T_{p1} - T_x)\{\theta_1 e^{-B} - B[\mathrm{Ei}(-B\beta) - \mathrm{Ei}(-B)]\}e^{\beta B}. \tag{14}$$

Fig. 2

Figure 2: Fig. 2

3rd stage—heating of the water in the chamber until eruption. The volume and pressure of the water in the chamber and the flow rate of the water in it do not change and are equal, respectively, to V_1, P_2, G_{x2}, G_2. Solving the heat-balance equation

$$\rho V_1 (dT/dt) = G_r (T_r - T) + G_{x2} (T_x - T)$$

with the initial conditions $t = t_2$, $T = T_2$, we obtain

$$T = T_{p2} - (T_{p2} - T_2) \exp[-(G_2/\rho V_1)(t - t_2)], \tag{15}$$

where

$$T_{p2} = (G_{x2} T_x + G_2 T_r)/G_2. \tag{16}$$

As the critical condition for a geyser eruption we take $T = T_{k2}$ (attainment of this temperature is possible when $T_{p2} > T_{k2}$). Then the expression for the induction period of a geyser eruption can be written in the form

$$t_{\text{ind}} = \frac{\rho(V_1 - V_0)}{G_1} + \frac{\rho V_k}{G_1 - G_2} \ln \frac{G_1}{G_2} + \frac{\rho V_1}{G_2} \ln \frac{T_{p2} - T_2}{T_{p2} - T_{k2}}. \tag{17}$$

By this formula, using expressions (16), (14), (13), (6), one can calculate t_{ind}, and also analyze the dependence of t_{ind} on various factors.

As an illustration, Fig. 2 gives the result of a calculation of the geyser regime for the following parameter values: $V_1 = 2 \cdot 10^3$ l, $V_0 = 1.4 \cdot 10^3$ l, $V_k = 10^3$ l, $h = 10$ m, $T_c = 200°$; $T_x = 75°$, $T_{k1} = 100°$, $T_{k2} = 120°$, $G_r = 1$ kg/sec; $G_{x1} = 5$ kg/sec, $G_{x2} = 0.5$ kg/sec.

Fig. 2

A geyser is an exceptional natural phenomenon; this means that it occurs within a narrow interval of parameter variation. Therefore any theory of the geyser process must also contain the conditions for the existence of a geyser (and of other related geothermal phenomena, such as hot springs and fumaroles). Analysis of the model considered has led to the following results.

$$T_{p1} < T_{k1} \begin{cases} T_{p2} > T_{k2} & \text{geyser} \\ T_{k1} < T_{p2} < T_{k2} & \text{boiling spring} \\ T_{p2} < T_{k1} & \text{hot spring} \end{cases}$$

$$T_{p1} > T_{k1} \begin{cases} c\left(T_{p1} - T_{k1}\right) > L & \text{fumarole} \\ c\left(T_{p1} - T_{k1}\right) < L \begin{cases} T_{p2} > T_{k2} & \text{geyser or pulsating boiling spring} \\ T_{p2} < T_{k2} & \text{boiling spring} \end{cases} \end{cases}$$

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Note: Figure translations are in progress. See original paper for figures.