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**Abstract**

**Full Text**

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## **THEORY OF ELASTICITY**

**I. V. KRAGELSKII, V. S. KOMBALOV**

# **CALCULATION OF THE MAGNITUDE OF STABLE ROUGHNESS AFTER RUNNING-IN (ELASTIC CONTACT)**

*(Presented by Academician A. Yu. Ishlinskii, January 13, 1969)*

With relative sliding of two solid bodies, a certain stable roughness is established on their contacting surfaces; it depends on the operating conditions of the pair and on their mechanical properties, and does not depend on the initial roughness<sup>(1,2)</sup>, etc.

Until now it has not been possible to determine the value of this roughness by calculation. The development of the molecular-mechanical theory of friction and of the fatigue theory of wear makes it possible to do this. According to<sup>(3-5)</sup>, the coefficient of friction is expressed by the three-term dependence

$$f = \tau_0/P_r + \beta + k_2(h/R)^{1/2}, \quad (1)$$

where  $\tau_0$  is the shear strength of the adhesive bond in the absence of load ( $\text{kg}/\text{mm}^2$ );  $P_r$  is the actual pressure at the contact ( $\text{kg}/\text{mm}^2$ );  $\beta$  is the coefficient of strengthening of the adhesive bond;  $h$  is the depth of penetration of rigid asperities of radius  $R$  into the deformable half-space;  $k_2$  is a coefficient taking into account the type of stress state and the geometry of the contact.

This widely tested formula<sup>(3-5)</sup> contains two special cases: the previously established two-term Derjaguin law of friction, which characterizes the adhesive component of friction, and the Grunwäle–Goryachkina law of friction—the last term of the formula, which characterizes the deformation component of friction. As the load  $P_r$ , and correspondingly the magnitude of penetration  $h$ , increase, the coefficient of friction passes through a minimum. On the other hand, according to the fatigue theory of wear<sup>(6,8)</sup>, the number of cycles  $n$  leading to failure is inversely proportional to the coefficient of friction  $f$ . As follows from formula (2), the smaller the coefficient of friction, the greater the number of cycles the material withstands, since in this case there will be smaller tensile stresses in the surface layer, which are responsible for the formation of wear particles. The number of cycles as a function of the acting stress is expressed

by the Wöhler curve <sup>(6,8)</sup>,  $n = (\sigma_0/kfP_r)^t$ , where  $\sigma_0$  is the extrapolated value of the destructive tensile stress at  $n = 1$ ;  $k$  is a coefficient characterizing the stress state and depending on the nature of the material;  $t$  is the exponent of the Wöhler fatigue curve. It is therefore natural that, in an ensemble of irregularities having different heights, those for which the coefficient of friction is smallest will be under the most favorable conditions. Therefore, on the run-in surface they will prevail.

Thus, the roughness of a surface having the minimum value of the coefficient of friction will correspond to the stable roughness.

Let us calculate this roughness, proceeding from a model in which a rigid surface, realized by a set of spherical irregularities of equal radius, slides over an elastically deformable smooth half-space. In doing so, the physicomaterial properties of this deformable material are taken into account.

The relative contact area  $\eta$ , as a function of the relative approach  $\varepsilon$ , is expressed by the power-law dependence <sup>(9)</sup>  $\eta = ab\varepsilon^\nu$ , where  $a$  is a coefficient depending on the stress and kinematic state of the contact (in the case of relative displacement for elastic contact  $a = 1/2$ );  $b$  and  $\nu$  are characteristics of the power-law dependence.

**Table 1**

Friction pair	Calculation				Experiment		
	$P_c, \text{ kg/mm}^2$	$\sigma_0, \text{ kg/mm}^2$	$E, \text{ kg/mm}^2$	$\Delta_{\text{calc}}$	$\Delta_{\text{exp}}$	$R_z, \mu$	Class
St. 45 — capron B	0.12	0.15	$1.5 \cdot 10^2$	$2.9 \cdot 10^{-2}$	$3 \cdot 10^{-2}$	1.2	9
St. 45 — fluoroplast-4	0.12	0.035	$7 \cdot 10^1$	$9 \cdot 10^{-3}$	$1 \cdot 10^{-2}$	2.64	8
St. 45 — poly- formalde- hyde	0.12	0.20	$1.7 \cdot 10^2$	$2.3 \cdot 10^{-2}$	$2.6 \cdot 10^{-2}$	1.15	9

On the other hand, the relative contact area in the case of elastic contact is expressed by a dependence of the form <sup>(9)</sup>

$$\eta = A_r/A_c = P_c/P_r = 1/2bh^\nu = 1/2bh^\nu/h_{\text{max}}^\nu, \quad (2)$$

where  $A_c$  and  $P_c$  are the contour contact area ( $\text{mm}^2$ ) and the corresponding contour pressure ( $\text{kg}/\text{mm}^2$ );  $\varepsilon = h/h_{\max}$  is the ratio of the indentation magnitude  $h$  (mm) to the maximum height of the irregularity  $h_{\max}$  (mm).

Expressing the quantity  $h$  from (2), we obtain:

$$h = (2P_c h_{\max}^\nu / bP_r)^{1/\nu}. \quad (3)$$

The actual pressure  $P_r$ , entering equations (1), (3), for the case of elastic contact, according to <sup>(9)</sup>, is equal to:

$$P_r = \left[ 2^{1/2\nu} k_2 h_{\max}^{1/2} E / (3/4\pi) b^{1/2\nu} R^{1/2} (1 - \mu^2) \right]^{2\nu/(2\nu+1)} P_c^{1/(2\nu+1)}. \quad (4)$$

Substituting into equation (1) the corresponding expressions (3) and (4), we obtain the coefficient of friction as a function of the roughness of the hard counterbody, the physicochemical properties of the deformable material, and the operating regime of the pair

$$f = A \left( \frac{h_{\max}}{Rb^{1/\nu}} \right)^{-\nu/(2\nu+1)} + \beta + B \left( \frac{h_{\max}}{Rb^{1/\nu}} \right)^{\nu/(2\nu+1)}, \quad (5)$$

where

$$A = \left[ \frac{3/4\pi(1 - \mu^2)}{k_2 E} \right]^{2\nu/(2\nu+1)} \frac{\tau_0}{(2P_c)^{1/(2\nu+1)}}, \quad B = \left[ \frac{2 \cdot 3/4\pi P_c (1 - \mu^2)}{E} \right]^{1/(2\nu+1)} k_2^{2\nu/(2\nu+1)}.$$

In the case considered,  $A$  and  $B$  are constants.

The roughness magnitude is estimated by the dimensionless ratio  $h_{\max}/Rb^{1/\nu}$ , characterizing the degree of sharpness of the asperities and their distribution by height, where  $R$  is the mean radius of rounding of the microroughnesses.

Denoting  $h_{\max}/Rb^{1/\nu} = \Delta$ , we obtain

$$f = A\Delta^{-\nu/(2\nu+1)} + \beta + B\Delta^{\nu/(2\nu+1)}. \quad (6)$$

The minimum value of the coefficient of friction will correspond to

$$df/d\Delta = 0, \quad d^2f/d\Delta^2 > 0. \quad (7)$$

Taking condition (7) into account, we obtain the following expression for the roughness corresponding to the minimum value of the coefficient of friction:

Fig. 1. Dependence of the friction coefficient of the pair steel 45–capron B on the roughness of the steel counterbody

Figure 1: Fig. 1. Dependence of the friction coefficient of the pair steel 45–capron B on the roughness of the steel counterbody

$$\Delta_{\text{calc}} = \left[ \frac{3/4\pi(1-\mu^2)}{E} \right]^{(2\nu-1)/2\nu} \tau_0^{(2\nu+1)/2\nu} \frac{1}{(2P_c)^{1/\nu} k_2^2} \quad (8)$$

Taking  $\nu = 2$  for run-in surfaces, we obtain the following simple expression, determining the dimensionless value corresponding to the stable roughness:

$$\Delta_{\text{calc}} = 2.7 \tau_0^{5/4} \left[ \frac{(1-\mu^2)}{E} \right]^{3/4} \frac{1}{P_c^{1/2}}, \quad (9)$$

where  $E, \mu$  are the elastic modulus and Poisson's ratio of the deformable material.

Experimental verification of the obtained calculated expression (8) was carried out for various metal–polymer pairs. The polymer specimens were run in against metallic specimens with different initial roughness. In this case the loads on the pair corresponded to conditions of elastic deformation at the contact. The sliding velocity was  $V = 0.5$  m/min; the temperature in the contact zone remained unchanged, which made it possible to judge the constancy of the physico-mechanical properties of the friction pair. The experiments were conducted without lubrication.

**Fig. 1.** Dependence of the friction coefficient of the pair steel 45–capron B on the roughness of the steel counterbody

The friction force reading was recorded; a profilogram was taken from the metallic surface, on the basis of which the values  $h_{\text{max}}, R, b$  and  $\nu$ , as well as the dimensionless ratio  $\Delta$  and the value  $R_z$ , were determined, which made it possible to estimate the surface-finish class. On the basis of the results obtained, an experimental dependence was constructed for the friction coefficient on the roughness of the run-in surface, determined by the dimensionless ratio  $\Delta$  (Fig. 1).

Proceeding from the contact conditions ( $\tau_0$  and  $P_c$ ), as well as the physico-mechanical properties of the deformable material, the value of the stable roughness was calculated by formula (8). In doing so,  $\mu = 0.4$ ;  $k_2 = 0.8$  were adopted. The values of  $\nu$  for the processed profilograms lie within the range 1.8–2.2.

Table 1 gives calculated and experimental data for the roughness of the metallic surface corresponding to the minimum value of the friction coefficient. The calculation results agree satisfactorily with the experiment.

Figure 1 presents the experimental dependence of the friction coefficient of the steel–capron B pair on the roughness of the rigid counterbody, illustrating the minimum of the friction coefficient.

Calculation by formula (8) of experimental data from a number of authors (<sup>10,11</sup>) also gave satisfactory agreement.

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