ON THE APPLICATION OF THE METHOD OF FAST MATRIX MULTIPLICATION TO THE PROBLEM OF FINDING THE TRANSITIVE CLOSURE OF A GRAPH

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Abstract

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MATHEMATICS

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ON THE APPLICATION OF THE METHOD OF FAST MATRIX MULTIPLICATION TO THE PROBLEM OF FINDING THE TRANSITIVE CLOSURE OF A GRAPH

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This algorithm makes it possible to find the transitive closure of a graph in $cn^{\log_2 7} \lg n$* operations, which is asymptotically better than the previously known estimate $cn^3/\lg n$ ([1]).

Lemma. *Boolean $n \times n$ matrices can be multiplied in $cn^{\log_2 7}$ operations.*

Indeed, we shall regard these matrices as arithmetic ones consisting of zeros and ones, and multiply them by the method ([2]) in $cn^{\log_2 7}$ operations. Then, in cn^2 operations, we inspect the elements of the product and replace every nonzero element by one. It is obvious that in this way we obtain the Boolean product of the matrices.

Theorem. *The transitive closure of a graph of order n can be obtained in $cn^{\log_2 7} \lg n$ operations.*

Proof. Let A_0 be the adjacency matrix of the given graph. We shall obtain the adjacency matrix of the transitive closure by the following iterations: $A_{n+1} = A_n \vee A_n^2$.

One iteration requires $cn^{\log_2 7}$ operations, and the number of iterations required is $\lg K$, where K is the maximum distance between two vertices (the diameter of the original graph), since each iteration halves the diameter of the graph. But $K \leq n$; thus, we required fewer than $cn^{\log_2 7} \lg n$ operations. This method is more advantageous than the method ([1]) for values of n on the order of 10^{19}.** However, if better estimates are obtained for the number of operations required for matrix multiplication, then this algorithm will also become advantageous for realistic n.

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REFERENCES

[1] V. L. Arlazarov, E. A. Dinitz, M. A. Kronrod, I. A. Faradzhev. DAN, 194, No. 3 (1970).
[2] V. Strassen, *Numerische Math.*, **13**, No. 4, 354 (1969).

* $\lg x$ is the least integer greater than or equal to $\log_2 x$; n is the number of vertices of the graph.
** With equal constants entering as factors in the estimate of the number of operations.

Note: Figure translations are in progress. See original paper for figures.