ON THE RADIAL STABILITY OF A CYLINDRICAL CONDUCTOR IN THE FIELD OF A TRAVELING MAGNETIC WAVE

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Fig. 1

Figure 1: Fig. 1

Abstract

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PHYSICS

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ON THE RADIAL STABILITY OF A CYLINDRICAL CONDUCTOR IN THE FIELD OF A TRAVELING MAGNETIC WAVE

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We consider the problem of the forces acting on an infinite cylindrical incompressible conductor of circular cross section in the quasi-stationary magnetic field of a surface wave of azimuthal current $i_\varphi = i_0\delta(R-b)\exp(i\omega t - ihz)$. The conductor is located inside the current surface $R = b$, eccentrically with respect to it (Fig. 1). The magnitude of the eccentricity d is arbitrary.

The required force, equal to the momentum flux of the electromagnetic field into the conductor, in the quasi-stationary approximation is completely determined by the value of the magnetic field on its surface

$$F_i = \frac{1}{4\pi}\oint\left(H_{iH}k - \frac{1}{2}\delta_{ik}H^2\right)n_k\,ds. \tag{1}$$

Fig. 1

Here n_k is the component of the unit vector of the external normal to the conductor surface ds, and summation is carried out over repeated indices. In finding the magnetic field we shall assume that the thickness of the skin layer is small compared with the radius of the conductor a, so that one may use the Leontovich boundary condition and solve only the exterior boundary-value problem with respect to the conductor ([1]).

We represent the magnetic field in the space between the current and the cylinder as the sum of the primary field of the currents in the empty system $\mathbf{H}^{(0)}$ and the field caused by the presence of the conductor $\mathbf{H}^{(1)}$. The longitudinal component of the primary field is equal to (the wave factor is omitted)

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

$$H_z^{(0)}(R) = BI_0(hR),$$

where

$$B = \frac{4\pi i_0}{c} hbK_1(hb),$$

and I and K are Bessel functions of imaginary argument.

Using the addition theorem for Bessel functions ([2]), $H_z^{(0)}$ may be written as a function of the polar coordinates r, φ, associated with the center of the conductor cross section,

$$H_z^{(0)}(r, \varphi) = B \sum_{n=-\infty}^{\infty} (-1)^n I_n(hd) I_n(hr) \cos n\varphi.$$

We shall seek the magnetic field of the currents induced in the conductor in the form of a series

$$H_z^{(1)}(r, \varphi) = B \sum_{n=-\infty}^{\infty} (-1)^n C_n K_n(hr) \cos n\varphi.$$

To find the coefficients C_n we have the Leontovich condition on the surface of the conductor $r = a$. It can be represented in the form

$$\left.\frac{\partial H_z}{\partial r}\right|_{r=a} = \zeta \frac{c}{v} \left.\left(H_z - \frac{1}{h^2 r^2} \frac{\partial^2 H_z}{\partial \varphi^2} \right)\right|_{r=a},$$

Fig. 2

Fig. 3

where v is the phase velocity of the wave, and $\zeta = \zeta' + i\zeta''$ is the surface impedance of the conductor, which in the case of sufficiently high conductivity is related to the skin-layer thickness δ by the formula $\zeta = \dfrac{1+i}{2} \dfrac{v}{c} \delta h$.

The final expressions for the components of the total magnetic field on the surface of the conductor have the form

$$H_z(a,\varphi) = -B\sum_{n=-\infty}^{\infty}(-1)^n\frac{I_n(\bar d)}{\bar a K_n'(\bar a)}\left[1 - i\zeta\frac{c}{v}\left(1+\frac{n^2}{\bar a^2}\right)\frac{K_n(\bar a)}{K_n'(\bar a)}\right]\cos n\varphi,$$

$$H_\varphi(a,\varphi) = iB\sum_{n=-\infty}^{\infty}(-1)^n\frac{nI_n(\bar d)}{\bar a^2 K_n'(\bar a)}\left[1 - i\zeta\frac{c}{v}\left(1+\frac{n^2}{\bar a^2}\right)\frac{K_n(\bar a)}{K_n'(\bar a)}\right]\sin n\varphi,$$

$$H_r(a,\varphi) = -\zeta\frac{c}{v}B\sum_{n=-\infty}^{\infty}(-1)^n\frac{I_n(\bar d)}{aK_n(\bar a)}\left(1+\frac{n^2}{\bar a^2}\right)\cos n\varphi,$$

$$\bar d = hd, \qquad \bar a = ha.$$

Substituting these expressions into the general formula (1), we first find the transverse force directed along the line connecting the axis of the cord with the axis of the current chamber.

In the calculation per unit length, the transverse force averaged over a period is equal to

$$F_\perp = F_{\perp 0} + F_{\perp\zeta},$$

$$F_{\perp 0} = 2\pi a P_0 G_0, \qquad F_{\perp\zeta} = -\pi a P_0 \zeta''\frac{v}{c}G_\zeta, \tag{2}$$

where

$$P_0 = \frac{B^2}{8\pi}, \qquad G_0 = \sum_{n=0}^{\infty}\Phi_n,$$

$$G_\zeta = \sum_{n=0}^{\infty}\frac{\Phi_n}{\bar a}\left\{\frac{\bar a^2 - n(n+1)}{\bar a^2 + n(n+1)} - \left(1+\frac{n^2}{\bar a^2}\right)\frac{\bar a K_n(\bar a)}{K_n'(\bar a)} - \left[1+\frac{n(n+1)}{\bar a^2}\right]\frac{\bar a K_{n+1}(\bar a)}{K_{n+1}'(\bar a)}\right\},$$

$$\Phi_n = \left[1+\frac{n(n+1)}{\bar a^2}\right]\frac{I_n(\bar d)I_{n+1}(\bar d)}{\bar a^2 K_n'(\bar a)K_{n+1}'(\bar a)}.$$

Here the direction toward the center of the chamber is taken as positive. The quantity $F_{\perp 0}$ corresponds to an ideal conductor, while $F_{\perp\zeta}$ is the correction for finiteconductivity. From (2) it is clear that $F'_{\perp 0} \geqslant 0$, i.e., the symmetric position of the cord is stable. For small deviations ($d \ll a$) this stability was shown in ([3]). Figure 2 presents curves of the dependence of the function G_0 on the magnitude of the ratio d/a, obtained as a result of numerical summation of the series. The parameter is the radius of the conductor. The linearity existing at small eccentricities is violated as d/a increases, the more rapidly the larger $\bar{a}$ is, which is connected with the increase of the magnetic field as the boundary of the conductor approaches the current surface.

The negative correction $F'_{\perp\xi}$ worsens the stability of the cord. Figure 2 shows the dependence of the function G_ξ on the magnitude of the deviation and on the radius of the cord.

In addition, the nonideality of the conductor leads to the appearance of a longitudinal force dragging the conductor in the direction of wave propagation,

$$F_{\parallel} = 4\pi\zeta'' \frac{c}{\omega} P_0 \sum_{n=0}^{\infty}{}' \left(1 + \frac{n^2}{\bar{a}^2}\right) \frac{I_n^2(\bar{d})}{\bar{a}[K'_n(\bar{a})]^2}.$$

The prime on the summation sign means that its zero term must be taken with coefficient 1/2. This expression makes it possible to judge the Joule losses in the conductor Q, whose magnitude is related to $F_{\parallel}$ by the formula

$$Q = vF_{\parallel}.$$

Figure 3 shows the dependence of $F_{\parallel}$ on the eccentricity of the system and on the radius of the conductor. The quantity $4\pi\zeta'' \frac{c}{\omega} P_0$ is taken as the unit of force. It is clear from the graphs that asymmetry of the system causes additional energy losses in the conductor.

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