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HYDROMECHANICS

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Abstract

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HYDROMECHANICS

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FLOW OF A SUPERSONIC STREAM OF ELECTRICALLY CONDUCTING GAS IN A NONUNIFORM MAGNETIC FIELD

1°. Let there be, in a plane channel of constant width d with dielectric walls, a supersonic ($\mathrm{M} > 1$) uniform gas flow with parameters $\rho_1, T_1, u_1, v_1 = 0$. The gas has finite electrical conductivity σ. At the initial instant $t = 0$ an external magnetic field $\mathbf{H}$ appears. Assuming the magnetic Reynolds number to be small, we shall regard the magnetic field H_z as a prescribed function of the coordinate x. In this case either

$$H_z = H_0 e^{\alpha x},\ x \leqslant 0; \qquad H_z = H_0,\ x > 0, \tag{1}$$

or

$$H_z = H_0,\ x \leqslant 0; \qquad H_z = H_0 e^{-\alpha x},\ x > 0. \tag{2}$$

In the first case we shall speak of "entry" into the magnetic field, in the second of "exit" from it.

The problem is described by a system of equations of unsteady magnetohydrodynamics. The electrodynamics is described by the equations $\operatorname{div} \mathbf{j} = 0$, $\operatorname{rot} \mathbf{E} = 0$, and Ohm' s law

$$\mathbf{j} = \sigma \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{H} \right).$$

The gas-dynamic part is described by the equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0;$$

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$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial}{\partial x}\left(p+\rho u^2\right)+\frac{\partial}{\partial y}(\rho uv)=\frac{1}{c}j_yH_z;$$

$$\frac{\partial(\rho v)}{\partial t}+\frac{\partial}{\partial x}(\rho uv)+\frac{\partial}{\partial y}\left(p+\rho v^2\right)=-\frac{1}{c}j_xH_z; \tag{3}$$

$$\frac{\partial}{\partial t}\left[\rho\left(\varepsilon+\frac{u^2+v^2}{2}\right)\right]+\frac{\partial}{\partial x}\left[\rho u\left(\varepsilon+\frac{p}{\rho}+\frac{u^2+v^2}{2}\right)\right]+$$

$$+\frac{\partial}{\partial y}\left[\rho v\left(\varepsilon+\frac{p}{\rho}+\frac{u^2+v^2}{2}\right)\right]=j_xE_x+j_yE_y.$$

It was assumed that the medium is a nonviscous, non-heat-conducting compressible fluid for which the equation of state of an ideal gas is valid, $p=\rho RT$, $\varepsilon=c_vT$. The electrical conductivity was taken either to be constant, $\sigma=\sigma_0=$ const, or

$$\sigma=\sigma_0\left(\frac{T}{T_1}\right)^{1/4}\left(\frac{\rho}{\rho_1}\right)^{-1/2}\exp\left[-\frac{I}{2}\left(\frac{1}{T}-\frac{1}{T_1}\right)\right]. \tag{4}$$

Here I is the ionization potential. It was also assumed that the Hall effect is absent.

In setting the boundary conditions for the gas dynamics, it was assumed that at the inlet the flow is supersonic ($\mathrm{M}_1=2.92$) and $\rho_1, T_1, u_1, v_1=0$ are prescribed. In the exit section no boundary condition need be specified if the flow remains supersonic ($\mathrm{M}>1$). If, however, in the course of the interaction at some instant $t=t_0$, regions appear in the exit section in which $\mathrm{M}=1$, then we shall assume that the value $\mathrm{M}=1$ remains there for any $t>t_0$. On the walls the condition of nonpenetration of the fluid, $v=0$, is used. The boundary condition in the electrodynamic part is the requirement of nonpenetration of the electric current on the entire boundary of the flow region.

2°. The posed problem was investigated analytically in [1, 2] under the assumption that the parameter of magnetohydrodynamic interaction

$$R_M=\sigma_0u_1dH^2/2c^2(p_1+\rho_1u_1^2)\ll 1.$$

However, the approximation $R_M\ll 1$ is not always justified. Then the nonlinear problem is described by equations (3). In the present work its solution was obtained by numerical methods. The equations of gas dynamics were integrated by a predictor-corrector scheme

Fig. 1

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Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Fig. 2. 1–$\alpha = \infty$, $\sigma = \text{const}$;
2–$\alpha = 3$, $\sigma = \text{const}$; 3–$\alpha = 3$, $\sigma \neq \text{const}$; 4–$\alpha = 10$, $\sigma \neq \text{const}$.
Dashed line–model solution, $M_1 = 2.92$, $\gamma = 1.12$

(3) with the use of the locally one-dimensional method [4] and matrix sweep in the implicit part. A special feature is the solution, at each time layer, of the stationary electrodynamic problem, which was carried out by iterative methods.

3°. Let us consider entry into a magnetic field increasing according to law (1). Figure 1 presents the distribution of the quantities ρ, T, and p, referred to their values at the channel inlet, of the Mach number M, and also shows the vector field of the current-density $\mathbf{j}$. The conductivity is $\sigma = \sigma_0 = \text{const}$, $\alpha = \infty$, $R_M = 1$. The calculations show that, when the magnetic field increases sharply ($\alpha > 10$), deceleration of the flow by the Lorentz force $\frac{1}{c}\mathbf{j} \times \mathbf{H}$ occurs mainly in a compression wave localized in the narrow zone $0 < x < \delta$, where the electric current I, induced for $x > 0$, is generated.

Under the assumption $\delta \ll d$, a model problem of the entry of a supersonic flow into the region of a uniform magnetic field was considered. Using

the known solution for $R_{\mathrm{M}} \ll 1$ ([1,2]) and writing the integral relations for the compression wave, values were obtained for the total induced current I, the number M_2 behind the compression wave, etc., which agree well with the numerical solution of the two-dimensional problem.

As follows from Fig. 1, the action of the Lorentz force leads to the effect of two-dimensional compression of the core of the flow toward the channel axis in the region of the compression wave. Therefore, the flow here proceeds as if in a converging channel with energy extraction. A steady transition through the speed of sound ([5]) in this case can occur only in a shock wave. Consequently, if behind the wave $\mathrm{M}_2 < 1$, then the structure of the compression wave contains a normal shock.

Consideration of the entry of the flow into a smoothly increasing field ($a = 3 \div 5$) shows that in this case the region of the compression wave extends over a distance comparable with the channel width, and rarefaction zones appear near the walls.

Figure 2 presents the variation of the minimum value of the Mach number $\mathrm{M}_{\min}$ as a function of R_{M}. For comparison, the result of solving the model problem

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Fig. 3

Figure 3: Fig. 3

for $\sigma = \text{const},\ a = \infty$ is also shown (dashed curve).

4°. When the flow of a conducting gas exits the magnetic field, deceleration and heating of the gas also occur as it interacts with the current vortex. However, the depth of penetration of the current vortex into the uniform magnetic field now always has the order of the channel width. This is due to the negative velocity gradient of the core of the flow at $x < 0$. The action of the Lorentz force pushes the gas away from the channel axis toward the walls, and the current filaments of the gas in the core of the flow expand.

Fig. 3

Figure 3 presents the results of the numerical solution of the problem for $R_{\mathrm{M}} = 1,\ a = 10$ and the conductivity law (4). The calculations showed that, when the conducting flow exits the magnetic field, a steady stationary flow regime with a smooth transition through the speed of sound ([5]) is realized (for sufficiently large R_{M}). This constitutes a qualitative difference between the flow at the exit from the field and at the entrance into it.

The influence of the parameter R_{M} on the degree of deceleration of the flow is shown in Fig. 4. The dashed line indicates the minimum value of M in the exit section $x = 1$. At a certain value $R_{\mathrm{M\,cr}}$, $\mathrm{M} = 1$ is reached in the exit section. A further increase of R_{M} does not change the value of M at $x = 1$. It follows from this that the entropy increase does not depend on R_{M} for $R_{\mathrm{M}} > R_{\mathrm{M\,cr}}$. But in a shock-free transition the entropy increase occurs only due to the release of Joule heat. Consequently, the integral release of Joule–

the heat release Q in the flow reaches a maximum at $R_{\mathrm{m\,cr}}$ and does not depend on R_{m} for $R_{\mathrm{m}} > R_{\mathrm{m\,cr}}$. The constancy of Q for $R_{\mathrm{m}} > R_{\mathrm{m\,cr}}$ is due to the division of the current vortex into two. As R_{m} increases, the left of them moves upstream.

5°. On the basis of the solution of the problem, the following conclusions may be drawn:

1. At finite values of the parameter of magnetohydrodynamic interaction R_{m}, a substantial rearrangement of the supersonic flow and of the electric-current field may occur.

2. The degree of flow deceleration and its nonuniformity in the transverse section increase with increasing R_{m}. There exists a critical value $R_{\mathrm{m\,cr}}$, exceeding which leads to a qualitative rearrangement of the flow into a subsonic one: at the entrance to the magnetic field a shock wave moves upstream, while at the exit a splitting of the current vortex into two is observed. In this case, the value of the integral Joule heating is established

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Fig. 4

Figure 4: Fig. 4

asymptotically with respect to R_{m}.

Fig. 4. 1—$a = 3,\ \sigma = \mathrm{const}$; 2—$a = 3,\ \sigma \neq \mathrm{const}$; 3—$a = 10,\ \sigma \neq \mathrm{const}$

3. A smooth profile of variation of the magnetic field reduces the degree of flow deceleration, but worsens its uniformity over the section.
4. There is a significant pressure and velocity gradient along the wall, which may affect the flow in the boundary layer.
5. A strong dependence $\sigma(\rho, T)$ promotes an increase in the effective value of the parameter R_{m} and leads to greater deceleration of the flow.

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