ON THE BEHAVIOR OF SOLUTIONS OF ELLIPTIC EQUATIONS NEAR THE BOUNDARY OF A DOMAIN OF TYPE \(A^{(1)}\)

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Abstract

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MATHEMATICS

B. N. KHIMCHENKO

ON THE BEHAVIOR OF SOLUTIONS OF ELLIPTIC EQUATIONS NEAR THE BOUNDARY OF A DOMAIN OF TYPE $A^{(1)}$

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Let, in a closed bounded domain $(T + \partial T)$ of n-dimensional Euclidean space R^n, a function $u(M)$ be defined which is a regular solution of the equation

$$\sum_{i,k=1}^{n} a_{ik}(M)\frac{\partial^2 u}{\partial x_i \partial x_k} + \sum_{i=1}^{n} b_i(M)\frac{\partial u}{\partial x_i} + c(M)u = f(M).$$

Here the functions a_{ik} are bounded in T, and

$$\sum_{i,k=1}^{n} a_{ik}\lambda_i\lambda_k \geq \alpha \sum_{i=1}^{n} \lambda_i^2, \qquad \alpha > 0.$$

Let further l be a ray issuing from the considered point M_0 of the boundary ∂T, such that $\cos(l, n) > 0$, where n is the inner normal to the surface ∂T at the point M_0.

Denote by $\varphi(\rho)$ a function satisfying the conditions:

$$\varphi(\rho) \in C^{(1)}([0, \rho_0]) \cap C^{(\infty)}((0, \rho_0]); \tag{1}$$

$$\varphi(0) = \varphi'(0) = 0; \tag{2}$$

$$\varphi'(\rho) > 0, \qquad \varphi''(\rho) > 0; \tag{3}$$

$$\int_0^{\rho_0} \frac{\varphi(t)\, dt}{t^2} < \infty. \tag{4}$$

We shall call a φ-paraboloid (see ([1])) the body $z_0 \geq z \geq \varphi(\rho)$. By A^*_φ and $A_{\varphi*}$ we denote the classes of domains from $A^{(1)}$ each boundary point of which can be touched by a φ-paraboloid, respectively from outside and from inside.

The construction of barriers for $u(M)$, analogous to those considered in ([2]), leads to the proof of the following theorems. It is assumed here that the maximum principle is valid.

Theorem 1. *If $(T + \partial T) \in A^*_{\varphi*}$ and at the point $M_0 \in \partial T$ $u(M)$ attains its minimum value u_0, then for every ray l there exists a constant $c_1 > 0$ such that, for $M \in l$ in a neighborhood of M_0,*

$$u(M) - u_0 \geq c_1 r_{10}.$$

Here (and below) r_{10} is the distance from M to M_0.

Denote by Ω^*_φ the body $z \geq -\varphi(\rho)$, $r \leq r_0$ $(r = \sqrt{z^2 + \rho^2})$, where $\varphi(\rho)$ satisfies conditions (1)–(3), but

$$\int_0^{\rho_0} \frac{\varphi(t)\,dt}{t^2} = \infty. \tag{5}$$

Theorem 2. If the function $u(M)$ is defined in Ω^*_φ and at the origin of coordinates M_0 attains its minimum value u_0, then for each ray l there exists a constant $c_2 > 0$ such that, for $M \in l$ in a neighborhood of M_0,

$$u(M) - u_0 \geq c_2 r_{10} \int_{r_{10}}^{r_0} \frac{\varphi(t)\,dt}{t^2}.$$

Let now $d(M)$ be the distance from the point M to the boundary ∂T, and suppose the functions $b_i(M)$, $c(M)$, and $f(M)$ everywhere in T satisfy the conditions

$$|b_i(M)| < c_3; \qquad |c(M)| d^\lambda(M) < c_3; \qquad |f(M)| d^\lambda(M) < c_3 \tag{6}$$

$$(0 < \lambda < 1)$$

and, moreover, on the surface ∂T the function $u(M)$ satisfies the requirement

$$\left| u(M) - u_0 - \sum_{i=1}^{n-1} a_i x_i \right| \leq c_4 \varphi(r_{10}), \tag{7}$$

where $u_0 = u(M_0)$, $a_i = \partial u(M_0)/\partial x_i$, and for the function $\varphi(\rho)$ conditions (1) –(4) hold.

Theorem 3. If $u(M)$ is defined in $(T+\partial T) \in A^*_\varphi$ and M_0 is an arbitrary fixed point of the boundary ∂T, then for each ray l there exists $c_5 > 0$ such that in a neighborhood of M_0

$$|u(M) - u_0| \leq c_5 r_{10}.$$

For harmonic functions, in the case where $(T+\partial T) \in A^*_\varphi \cap A_{\varphi *}$, the assertion of Theorem 3 follows directly from the results given in [3].

Denote by $\Omega_{\varphi *}$ the φ-paraboloid for which $\varphi(\rho)$ satisfies conditions (1)–(3) and (5), and suppose that, for the function $u(M)$ defined in $\Omega_{\varphi *}$, on the surface $z = \varphi(\rho)$ one has

$$\left| u(M) - u_0 - \sum_{i=1}^{n-1} a_i x_i \right| \leq c_6 \varphi(r_{10}) \exp\left[-c_7 \int_{r_{10}}^{r_0} \frac{\varphi(t)\, dt}{t^2} \right],$$

where $u_0 = u(M_0)$, M_0 is the vertex of the φ-paraboloid, and c_6 and c_7 are fixed positive constants.

Theorem 4. For each ray l there exist c_8 and c_9 such that, in a neighborhood of M_0,

$$|u(M) - u_0| \leq c_8 r_{10} \exp\left[-c_9 \int_{r_{10}}^{r_0} \frac{\varphi(t)\, dt}{t^2} \right].$$

Moscow State University
named after M. V. Lomonosov

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REFERENCES

[1] B. N. Khimchenko, DAN, **192**, No. 1 (1970).
[2] B. N. Khimchenko, Differential Equations, **5**, No. 10 (1969).
[3] Kjell-Ove Widman, Math. Scand., **21**, 17 (1967).

Note: Figure translations are in progress. See original paper for figures.