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Abstract

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THEORY OF ELASTICITY

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ON THE LAWS OF REFLECTION OF BODY FORCES IN THE THEORY OF ELASTICITY THROUGH A PLANE BOUNDARY

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It is known that some boundary-value problems of the theory of elasticity can be solved by superposition of Kelvin solutions. These include the problems of Boussinesq, Cerruti (^{1}), and Mindlin (^{2}). The most general of them is Mindlin' s problem on the action of a concentrated force inside an elastic half-space whose boundary is free of external actions. In the present paper a further development and generalization of Mindlin' s method is given by realizing a simple and general idea. This idea is based on the analytic continuation of solutions of the equations of the theory of elasticity through a plane boundary. Knowledge of the laws of analytic continuation (reflection) makes it possible to construct solutions of boundary-value problems in a general form and to obtain reflection laws for body forces. The problem of analytic continuation through a plane boundary was considered in works (^{3,4}).

In work (^{4}), analytic continuation was used to solve problems of the theory of elasticity with mixed boundary conditions. Some of the results of that work are presented in the monograph (^{5}).

The main results of the present paper are formulated in the following theorem.

Theorem. *Let in the half-space* $x_3 > 0$*, on whose boundary* $x_3 = 0$ *either a) displacements or b) stresses are absent, body forces* $\dot{F}_i$ *act, with* $\dot{F}_i(x_1, x_2, x_3) = 0$ *for* $x < 0$ $(i = x_1, x_2, x_3)$*. Then the displacements that arise in the elastic half-space* $x_3 > 0$ *from the action of these forces can be regarded as the solution of an elastic problem for the whole space, in which the following system of body forces acts:*

$$\text{a)}\quad F_i = (\dot F_i - \dot F_i') + \frac{2(1-\nu)}{(1-2\nu)(3-4\nu)}\, x_3 \left[2\,\frac{\partial \dot F_3'}{\partial x_i} - \frac{1}{2(1-\nu)}\, x_3 \times \right.$$

$$\left. \times \frac{\partial}{\partial x_i} (\operatorname{div} \dot{\mathbf{F}})' \right] + \frac{2}{(1-2\nu)(3-4\nu)}\, \delta_{i3} \left[\dot F_3' - 2\nu x_3 (\operatorname{div} \dot{\mathbf{F}})' \right] \quad (i = x_1, x_2, x_3);$$

$$\text{b)}\quad F_i = (\dot F_i + \dot F_i') - \frac{4(1-\nu)}{1-2\nu}\, x_3 \frac{\partial \dot F_3'}{\partial x_i} + 4(1-\nu) \frac{\partial}{\partial x_i} \int_{x_3}^{+\infty} \dot F_3' \, dx_3$$

$$+ \frac{1}{1-2\nu} x_3^2 \frac{\partial}{\partial x_i} (\operatorname{div} \dot{\mathbf{F}})' + 2(1-\nu) x_3 \frac{\partial}{\partial x_i} \int_{x_3}^{+\infty} (\operatorname{div} \dot{\mathbf{F}})' \, dx_3$$

$$- 2(1-\nu) \int_{x_3}^{+\infty} x_3 (\operatorname{div} \dot{\mathbf{F}})' \, dx_3 \quad (i = x_1, x_2),$$

$$F_3 = \dot F_3 - \frac{1 + 2\nu(3-4\nu)}{1-2\nu} \dot F_3' - \frac{4(1-\nu)}{1-2\nu} x_3 \frac{\partial \dot F_3'}{\partial x_3} + \frac{4\nu}{1-2\nu} x_3 (\operatorname{div} \dot{\mathbf{F}})' + \frac{1}{1-2\nu} x_3^2 \frac{\partial}{\partial x_3} (\operatorname{div} \dot{\mathbf{F}})' + 2\nu \int_{x_3}^{+\infty} (\operatorname{div} \dot{\mathbf{F}})' \, dx_3.$$

Here ν is Poisson' s ratio, δ_{i3} is the Kronecker symbol, and the prime operation denotes replacing x_3 by $(-x_3)$. We note that the integrals may turn out to be divergent in the ordinary sense; therefore they should be understood in the sense of Hadamard or, equivalently, in the sense of the theory of generalized functions (^{6}).

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Note: Figure translations are in progress. See original paper for figures.

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