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ON A THEOREM OF M. V. KELDYSH AND M. A. LAVRENTIEV

MATHEMATICS

1970

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Abstract

Full Text

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MATHEMATICS

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ON A THEOREM OF M. V. KELDYSH AND M. A. LAVRENTIEV

(Presented by Academician A. N. Tikhonov on October 6, 1969)

The central result of paper ([1]) is Theorem 2, from which there immediately follows uniqueness (up to a constant) of the solution of the Neumann problem. This theorem is formulated as follows.

Let $u(M)$ be a function harmonic in some domain T of three-dimensional Euclidean space, not identically constant, and suppose that at a point M_0 of the boundary ∂T, $u(M)$ has a unique limiting value u_0, equal to the lower bound of its values in T.

If in T one can inscribe a body congruent to the paraboloid $z_0 \geq z \geq \rho^{1+\alpha}$ $(\alpha > 0;\ \rho = \sqrt{x^2 + y^2})$ with vertex at M_0, then

$$\lim_{r_{10} \to 0} \frac{u(M_1) - u_0}{r_{10}} > 0,$$

where M_1 is a point on the axis of the paraboloid, and r_{10} is the distance from M_1 to M_0.

Let us now introduce the function $\varphi(t)$ $(t \in [0, t_0])$, satisfying the following conditions:

$$\varphi(t) \in C^{(1)}([0, t_0]) \cap C^{(\infty)}((0, t_0]); \tag{1}$$

$$\varphi(0) = \varphi'(0) = 0; \tag{2}$$

$$\varphi'(t) > 0,\ \varphi''(t) > 0 \quad \text{in } (0, t_0); \tag{3}$$

$$\int_0^{t_0} \frac{\varphi(t)\, dt}{t^2} < \infty. \tag{4}$$

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We shall call the body $z_0 \geq z \geq \varphi(\rho)$ $\left(\rho = \left(\sum_{i=1}^{n-1} x_i^2\right)^{1/2}\right)$ a φ-paraboloid.

By constructing a lower barrier, one can prove that Theorem 2 remains valid in a domain $(T + \partial T)$ of n-dimensional Euclidean space whose boundary point M_0 can be touched from within by a φ-paraboloid. As such a barrier one may take the subharmonic function

$$\Psi_1(M) = z \exp\left[\lambda_1 \int_0^z \frac{\varphi(t)\,dt}{t^2}\right] - \lambda_2 \varphi(r),$$

where $r = \sqrt{\rho^2 + z^2}$, and λ_i (here and below, λ_i are positive constants).

At the same time, Theorem 2 fails in certain domains $(T + \partial T) \in A^{(1)}$.

Indeed, define a harmonic function $u(M)$ in a φ-paraboloid, where $\varphi(t)$ satisfies conditions (1)–(3), but

$$\int_0^{t_0} \frac{\varphi(t)\,dt}{t^2} = \infty.$$

Moreover, $u(M)$ must attain the minimal value u_0 in some neighborhood of M_0. Then on the axis Oz

$$u(M_1) - u_0 \leqslant \lambda_3 z \exp\left[-\lambda_4 \int_z^c \frac{\varphi(t)\,dt}{t^2}\right].$$

As an upper barrier here one proposes the superharmonic function

$$\Psi_2(M) = \exp\left[-\lambda_4 \int_r^{r_0} \frac{\varphi(t)\,dt}{t^2}\right] (\lambda_5 z - \varphi(r)).$$

I express my sincere gratitude to the participants of A. N. Tikhonov' s seminar for their attention to this work.

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REFERENCES

[1] M. V. Keldysh, M. A. Lavrent' ev, DAN, 16, No. 3 (1937).

Note: Figure translations are in progress. See original paper for figures.

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