ON THE STABILITY OF A CYLINDRICAL ORTHOTROPIC SHELL UNDER LONGITUDINAL IMPACT

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Abstract

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THEORY OF ELASTICITY

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ON THE STABILITY OF A CYLINDRICAL ORTHOTROPIC SHELL UNDER LONGITUDINAL IMPACT

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In recent years a number of works have been published devoted to the study of the behavior of closed cylindrical isotropic shells under rapid loading by an axial force and transverse pressure. A review of these works is given in ([1]).

In the present article we investigate the buckling of a cylindrical fiberglass shell, treated as orthotropic, under the action of an actual impact load. It is assumed that the shell has initial imperfections in the form of its middle surface.

Consider a circular cylindrical shell of mass M_1, fixed at one of its ends, under the assumption that an axial impact is applied at the free end by a load of mass M_2. Experiments carried out by the authors of the article have shown that the character of buckling of the shell is approximately the same as in the case of static loss of stability in the large. Therefore, as the basis of the problem we take the dynamic differential equations of the nonlinear theory of shells. Further, we assume that the deformations caused by impact are distributed along the length of the shell in the same way as in the case of a straight elastic rod of mass M_1. Integrating the corresponding wave equation for the longitudinal displacement u by the method of characteristics, we determine the law of variation in time of the inertial compressive stresses for different transverse sections of the shell. We then assume that this law of variation of the compressive stresses extends over a certain narrow zone near the section. For this zone we write the dynamic equations of the nonlinear theory of shells and integrate them, taking into account the previously found inertial components. We assume here that the initial and developing dents have a rhomboidal character. As a result of integrating the dynamic equations, we determine the dependence of the deflection on time. We suppose that the shell loses stability under impact if it snaps through to a new equilibrium form.

We shall proceed from the known nonlinear equations for orthotropic shells

$$\frac{1}{h}\nabla_D^4(w-w_0) = L(w,\Phi) + \frac{1}{R}\Phi_{,xx} - \frac{\gamma}{g}w_{,tt}, \tag{1}$$

where

$$\nabla_D^4 = D_1 \partial^4/\partial x^4 + 2D_3 \partial^4/\partial x^2 \partial y^2 + D_2 \partial^4/\partial y^4,$$

$$L(w, \Phi) = w_{,xx}\Phi_{,yy} + w_{,yy}\Phi_{,xx} - 2w_{,xy}\Phi_{,xy};$$

$$\delta_2 \Phi_{,xxxx} + 2\delta_3 \Phi_{,xxyy} + \delta_1 \Phi_{,yyyy} = -w_{,xx}w_{,yy} + w_{,xy}^2 + w_{0,xx}w_{0,yy} - w_{0,xy}^2 - \frac{1}{R}(w - w_0)_{,xx}, \tag{2}$$

where Φ is the stress function in the middle surface; w and w_0 are the total and initial deflections; t is time; h is the shell thickness; R is the radius of curvature. The coordinates x and y are measured along the generator of the shell and

along the arc. We approximate the functions w and w_0 by means of the expressions

$$w = f(\sin \alpha x \cdot \sin \beta y + \psi \sin^2 \alpha x \cdot \sin^2 \beta y), \tag{3}$$

$$w_0 = f_0(\sin \alpha x \cdot \sin \beta y + \psi \sin^2 \alpha x \cdot \sin^2 \beta y). \tag{4}$$

Here $\alpha = m\pi/L$; $\beta = n/R$; m is the number of half-waves along the generator of the shell; n is the number of waves around the circumference of the shell. Substituting (3) and (4) into (2), we find the function Φ; into the expression for Φ we introduce the term $(-py^2/2)$. For the dimensionless parameters and notation used in the work, see [2, 3].

Next, applying the usual Bubnov–Galerkin procedure to equation (1), we arrive at a system of ordinary differential equations

$$\frac{d^2\zeta}{d\hat{t}^2} - S_3 \left\{ \left[\frac{\hat{p}}{A_1} - \frac{A_2}{A_1}(\zeta^2 - \zeta_0^2) \right] \zeta - (\zeta - \zeta_0) - \frac{A_3}{A_1}(\zeta^3 - \zeta_0^2\zeta)\psi^2 - \frac{A_4}{A_1}(\zeta^2 - \zeta_0\zeta)\psi - \frac{A_5}{A_1}(\zeta^2 - \zeta_0^2)\psi \right\} = 0, \tag{5}$$

where

$$S_3 = A_1^3 \eta \xi^2 \left(\frac{h}{R}\right)^2 \left(\frac{L}{R}\right) \left(\frac{1}{v_0}\right)^2 \frac{g}{\gamma} \sqrt{E_1 E_2};$$

$$\hat{p} = \frac{p}{\sqrt{E_1 E_2}} \frac{R}{h};$$

A_1 is the parameter of the upper critical stress under static loading, determined by the formula:

$$A_1 = \lambda_3 \eta \vartheta / \xi^2 + \Delta \xi^2 \lambda_2 / \eta, \qquad \Delta = \sqrt{E_2/E_1}.$$

We take the expression for ψ from the solution of the static problem:

$$B_3' \zeta(\zeta^2 - \zeta_0^2)\psi^3 - B_4'(\zeta^2 - \zeta^2)\psi^2 - B_5'(\zeta - \zeta_0)\zeta\psi^2 - \hat{p}\zeta\psi + B_6'\zeta(\zeta^2 - \zeta_0^2)\psi + B_7'(\zeta - \zeta_0)\psi - B_1\zeta(\zeta - \zeta_0) - B_2'(\zeta^2 - \zeta_0^2) = 0. \tag{6}$$

In equations (5) and (6), the coefficients $A_2 \div A_5$ and $B_1' \div B_7'$ are quantities depending on the parameters of wave formation and on the mechanical characteristics of the glass-reinforced plastics.

Integration of equations (5) and (6) was carried out by the Runge–Kutta method using the BESM-2M electronic digital computer under the initial conditions

$$\zeta = \zeta_0, \qquad d\zeta/d\hat{t} = 0 \quad \text{at } \hat{t} = 0. \tag{7}$$

To investigate the influence of the degree of anisotropy of the shells on the character of their buckling, $E_1 E_2 = \text{const}$ was assumed. The ratio E_1/E_2 was then varied. In the calculations it was taken equal to 1, 2, and 5. In addition, for shells made of material P2-7S and AG-4S, other values of E_1/E_2 were adopted.

Figure 1 gives curves reflecting the dependence $\zeta_1(\hat{t})$ in the case $E_1/E_2 = 5$; $M_2/M_1 = 30$; $\xi = 2$ for a shell with parameters $\zeta_0 = 0.001$; $L/R = 2.2$; $h/R = 1/250$ and different initial impact velocities ($x = L/2$). From the curves constructed for different wave numbers n, the curve corresponding to the earliest buckling was selected ($n = 12$ in Fig. 1). The ratio of the initial impact velocity to the velocity of propagation of elastic waves in the shell material v_0/V was taken equal to 0.001, 0.015, and 0.020. Similar curves were obtained for the case of an infinitely large mass of the striking load. In addition, the parameter ξ, which determines the shape of the dent, was varied; calculations were carried out for shells with different geometric parameters.

Let us present the results of an experimental study of the behavior of fiberglass cylindrical shells under axial impact. The shells were made of P2-7S and AG-40 material by winding longitudinal and transverse layers of glass tape. Thirty-five specimens were tested. The test setup and procedure are described in paper (3). Figure 2 compares the experimental values of the coefficient k_d with the theoretical results (for the determination of k_d, see paper (1), p. 768). The curves in the figure show the dependence of the dynamic coefficient k_d on the initial impact velocity for shells with relative dimensions $L/R = 3.5$, $R/h = 7.2$, for the ratio $M_2/M_1 = 30$, $E_1/E_2 = 1$ and 2. The circles here indicate the values of the dynamic coefficient obtained from experiment. As is clear from

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

the graph, the theoretical and experimental data agree satisfactorily with one another.

Fig. 1. Dependence $\xi_1(\hat{t}')$ for $v_0 = 0.001\,v,\ n = 12$ (1), $v_0 = 0.015\,v,\ n = 13$ (2), and $v_0 = 0.020\,v,\ n = 14$ (3)

Fig. 2. Dependence of K_d on the initial impact velocity $((v_0/V) \cdot 10^{-3})$ for $E_1/E_2 = 1$ (1) and $E_1/E_2 = 2$ (2)

Hence it may be concluded that the presented solution of the problem of buckling of a fiberglass shell under axial impact is acceptable, at least in a first approximation.

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Note: Figure translations are in progress. See original paper for figures.