On the Solution of the Inverse Problem of Potential Theory by the Method of Fitting with the Aid of the Display Method

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On the Solution of the Inverse Problem of Potential Theory by the Method of Fitting with the Aid of the Display Method

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This paper describes a method for solving the inverse problem of potential theory in its variometric variant. The essence of the method is that the solution is specified very approximately (in the zero approximation) and is then refined with the aid of an analog device equipped with a display unit. To test this method, a real problem was solved (see Fig. 2). The solution was distinguished by high accuracy, despite the fact that obtaining it requires no special mathematical training. The method described may find application in the field of ore geophysics.

There are many interesting investigations on the inverse problem of potential theory; however, among the methods known to us only two are suitable for practical applications—the analytical methods of Grant ([1]) and Shalaev ([2]). They were developed for the gravimetric variant of the inverse problem of potential theory, in which one seeks the form of the surface S of a gravitating body T with constant excess density σ, if on a certain surface S_0 (the Earth' s surface) the values of $\partial V/\partial z$ of the vertical gradient of the potential V of the body T are known.

Here we shall consider this problem in a somewhat different form—the two-dimensional variometric variant of the inverse problem of potential theory, in which the unknown S is the contour of the cross section of an infinitely long cylinder with constant excess density σ, and the known quantity is the value of the horizontal gradient of gravity $\partial^2 V/\partial x\,\partial z$ on the line S_0, coinciding with the OX axis.

The difference in the two formulations is that in the first case, in some interval $[-a, b]$ (here and below we shall assume that $b > 0$, $a > 0$) $\partial V/\partial z|_{z=0}$ is specified, while in the second $\partial^2 V/\partial x\,\partial z|_{z=0}$ is specified.

In reality the problem is complicated by the presence of extraneous disturbing masses. The potential U of these masses plus the potential V of the body T give the total potential W

$$W = U + V. \tag{1}$$

Therefore, in order to reformulate the problem, it is necessary, starting from the values
$\partial^2 W/\partial x\,\partial z|_{z=0} = f(x),\ -a \le x \le b$, which are known from measurements, to find the function $\partial V/\partial z|_{z=0} = F(x),\ -a \le x \le b$.

As in other gravimetric investigations, we eliminate the extraneous field on the assumption that the contribution of the disturbing masses to the anomaly is equal to some constant α

$$\left.\frac{\partial^2 U}{\partial x\,\partial z}\right|_{z=0} = \alpha, \qquad -a \le x \le b. \tag{2}$$

To determine α, we integrate (1) over the interval $[-a, b]$

$$I = \int_{-a}^{b} \left.\frac{\partial^2 W}{\partial x\,\partial z}\right|_{z=0} dx = \int_{-a}^{b} \left[\left.\frac{\partial^2 V}{\partial x\,\partial z}\right|_{z=0} + \alpha \right] dx.$$

If we assume that the body is bounded and that the ordinate of the center of mass $z^* \ll a + b$, then, to a certain approximation, we have

$$\partial V/\partial z = kM/(z^{*2} + a^2) \quad \text{for } x = -a,\ z = 0;$$

$$\partial V/\partial z = kM/(z^{*2} + b^2) \quad \text{for } x = b,\ z = 0,$$

where M is the mass of the body per unit length.

Using these formulas, we obtain

$$\int_{-a}^{b} \left.\frac{\partial^2 V}{\partial x\,\partial z}\right|_{z=0} dx = \frac{kM}{z^{*2} + b^2} - \frac{kM}{z^{*2} + a^2},$$

$$\alpha = \frac{I}{a+b} + \frac{kM}{a+b} \left(\frac{1}{z^{*2} + a^2} - \frac{1}{z^{*2} + b^2} \right). \tag{3}$$

With the aid of the known formulas for determining M and z^* from the anomalous curve, we compute α, and then also the function

$$g(x) = \left.\frac{\partial^2 V}{\partial x\,\partial z}\right|_{z=0}, \qquad -a \le x \le b.$$

Now we can determine the values of the function $F(x)$ in the interval $-a \le x \le b$:

$$F(x) = \frac{M}{z^{*2} + a^2} + \int_{-a}^{x} g(x)\,dx, \qquad -a \le x \le b. \tag{4}$$

Thus the inverse variometric problem is reduced to the inverse gravimetric problem, which we shall consider below.

We obtain the solution of the two-dimensional gravimetric problem with the aid of a specialized analog computing device equipped with a display unit (Fig. 1). One of the important elements of this device is a two-layer circular electrolytic tank 1 ([3]). The electric field in the tank is produced by special supply electrodes.

A specific feature of the device is the configuration of the supply electrodes, which coincides with the contour S^* of the modeled body. The curve F^* corresponding to this contour is obtained from the potentials at a number of points of the straight line S_0^*, which imitates the Earth' s surface S_0.

By means of the electronic commutator 2, these potentials are fed sequentially to the screen of the oscilloscope 3, where a visual image F^* is obtained. On the same screen the anomaly F of the true body is also displayed visually. In the case where S^* coincides with the contour S of the true body, F^* would coincide with F. To achieve this, the device uses the display principle, whose essence in the present case is as follows. The human operator 4 selects some zero approximation of the model contour S_0^*, and on the screen of the oscilloscope 3 compares the corresponding curve F_0^* with the true anomaly F. From the nature of the discrepancy between F_0^* and F, he selects in the tank a new contour S_1^*, so that the discrepancy between S_1^* and S is reduced. This process proceeds through a sequence of model contours $S_0^*, S_1^*, S_2^*, ...$ and corresponding model anomalies $F_0^*, F_1^*, F_2^*, ...$, until a contour S^* is obtained in the tank whose anomaly F^* differs only insignificantly from the anomaly of the true body.

To test the described method, the problem was solved of interpreting the gravitational anomaly of a chromite ore vein, obtained from measurements with a variometer. From the gradient curve $f(x)$ (Fig. 119 of work ([4]), where $f(x)$ is line 1), using (4), the curve $F(x)$ was obtained, which was then interpreted on the model device. The process of finding successive approximations was stopped when it became impossible to further reduce the quantity

$$\psi = \max |F^*(x) - F(x)| / F_{\max}(x),$$

$-a \le x \le b$, whose minimum value is 0.042. The corresponding contour S^* is shown in Fig. 2. Comparison with the true contour S of the ore body shows the high accuracy of the method described. A more detailed study of the errors of the method will be published in a separate paper.

The process of finding successive approximations is accelerated if the operator uses certain rules of interpretation. Thus, for example, since the problem is ill-posed, one of the rules is to artificially suppress oscillating solutions. Another rule recommends choosing the initial approximation as a circle, whose radius and center are determined from comparison of the extreme portions of the curves F_0^* and F.

Fig. 1

Fig. 2

Fig. 1. Diagram of the analog device. 1 –electrolytic bath; 2 –commutator; 3 –oscilloscope; 4 –human operator

Fig. 2. Results of solving the inverse problem. S_0 –earth' s surface; S –contour of the ore body; S^* –contour of the model body. Error measure η = hatched area / area S

The idea of the device described here and of the device from work (5) is given in (6, 7). Technically these devices are similar, but in their operating principle they differ substantially. In the new device the display method plays a major role, and it is much more effective than the old one, in which problems are solved by means of a rigid algorithm.

If one disregards inessential differences, the new device is identical to the devices described in works (8-10).

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Note: Figure translations are in progress. See original paper for figures.